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Mathematics in the Public Schools
Curriculum Committee

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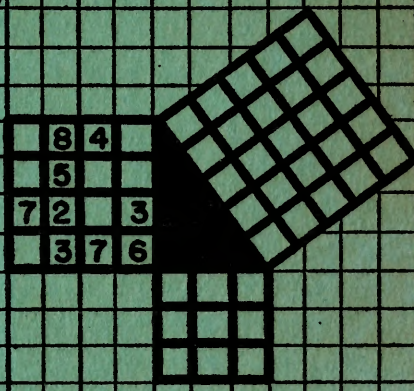
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MATHEMATICS



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NORTH CAROLINA PUBLIC SCHOOLS — YEARS 1-12



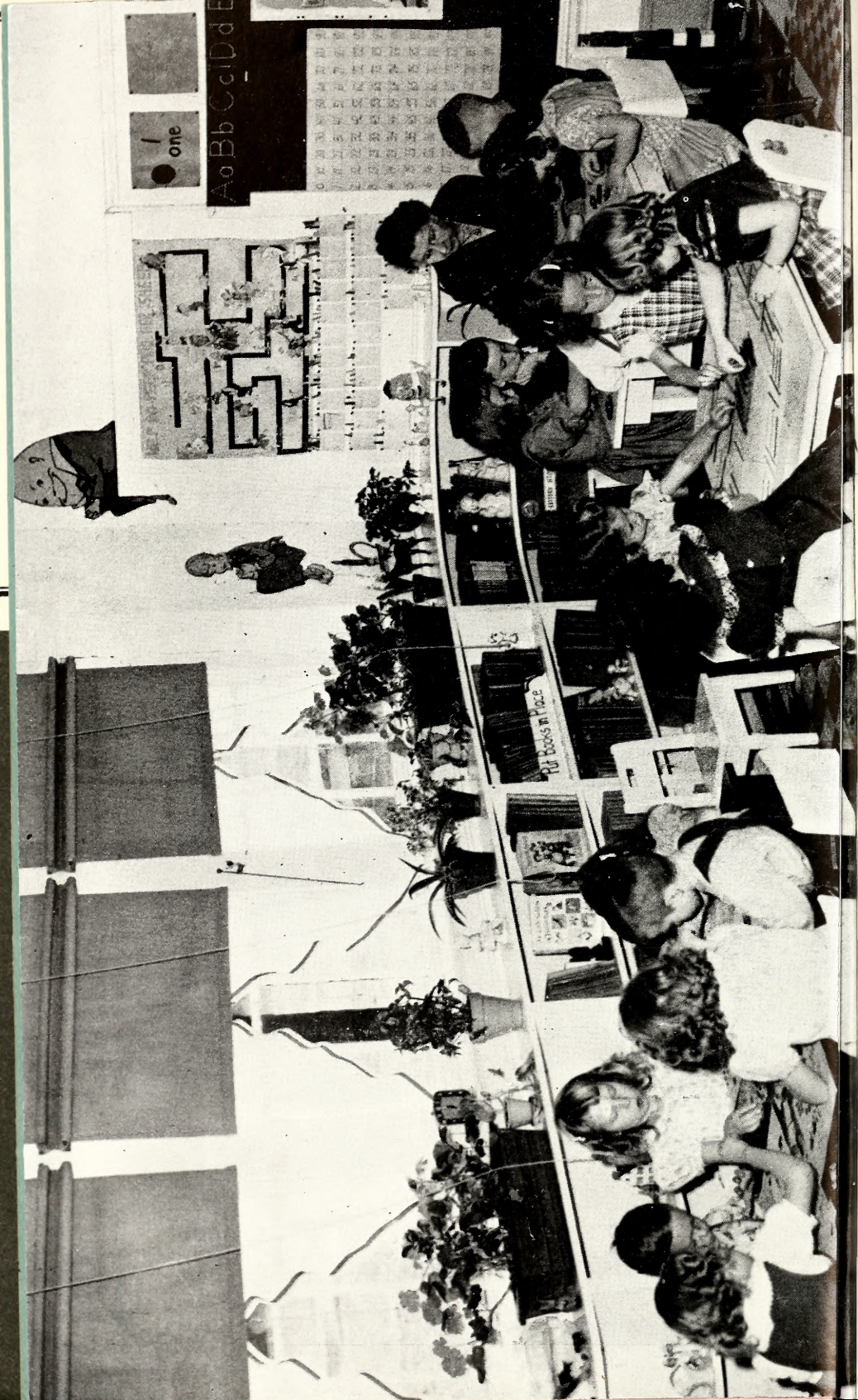
PUBLICATION No. 275

Mathematics in the Public Schools



Prepared by
THE MATHEMATICS CURRICULUM COMMITTEE
with the help of
ADMINISTRATORS AND TEACHERS IN NORTH CAROLINA

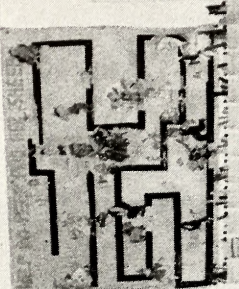
Issued by the
STATE SUPERINTENDENT OF PUBLIC INSTRUCTION
RALEIGH, NORTH CAROLINA



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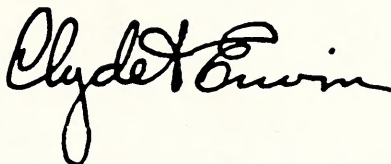
Put Books in Place

FOREWORD

Many children enter school with a well developed quantitative language for their age levels, but somewhere in their school experiences they get the idea that mathematics is a body of facts to be memorized; they are plunged into processes beyond their range of comprehension; and they develop an aversion to the subject. Aversion to mathematics and a lack of understanding go hand in hand. The lack of understanding of mathematics that is common to many children throughout the elementary and high school is a challenge to teachers to find meaningful, effective ways of teaching the subject. This bulletin was written to help teachers meet that challenge.

The suggestions in the bulletin were based on sound practices recommended by superintendents, supervisors, principals, and teachers throughout the State. The illustrations were taken from reports of actual classroom situations in North Carolina. The pictures in the bulletin were made in North Carolina schools. The suggested sequence in developing fundamental concepts, principles, and skills in mathematics was based on studies that have been made to show the levels at which children can understand certain mathematical processes. The sequence was outlined originally by the 1942 mathematics curriculum committee.

It is recommended that teachers get an overall view of the mathematics program from the first year through the twelfth. This publication will help teachers get such an overall view.



State Superintendent of Public Instruction

May 22, 1950

“Arithmetic is a system of quantitative thinking which we teach children, so that, as children and later as adults, they may live more efficiently, more intelligently, more richly, and more happily in our culture.”

DR. WILLIAM A. BROWNELL.

PREFACE

Because of the importance of mathematics in the school curriculum this revision of the program in arithmetic and high school mathematics has been prepared.

The work of the mathematics committee which prepared the outline for the 1942 publication, *A Suggested Twelve Year Program for the Public Schools of North Carolina*, has been carefully studied by many teachers throughout the State. The program as originally planned has stood up well. In each administrative unit a committee consisting of a primary, a grammar grade and a high school teacher was appointed and a number of conferences were held throughout the State. The conferences for the study of the program were arranged and conducted by Miss Mary Vann O'Briant and Mr. A. B. Combs. Mr. Henry A. Shannon rendered valuable assistance in the later stages of the work. In addition written comments were submitted by many teachers. The present revision seems to meet the approval of the vast majority of the teachers.

The illustrations used throughout the bulletin were taken from actual classroom situations and should be helpful in suggesting good practices.

In addition to the new material and to the careful check which was made of the outlines by years, additions have been made to the bibliographies for both elementary and secondary school teachers. From the bibliographies it should be possible to provide a small collection of books, magazines and materials which will be of considerable assistance to teachers.

This bulletin is an attempt to make available to every teacher in usable form suggestions which we believe will result in improvement in the teaching of mathematics.

J. HENRY HIGHSMITH, *Director*
Division of Instructional Service

ACKNOWLEDGMENTS

The following curriculum committee wrote the section on mathematics in the publication, *A Suggested Twelve Year Program for the North Carolina Public Schools*:

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Dr. William Betz, *Consultant*

A State-wide curriculum committee, made up of three people from every administrative unit, continued the work of the original committee. To facilitate travel and to make good working situations, this Mathematics Curriculum Committee was divided into sixteen groups, with sub-divisions when needed. This bulletin is a revision of the material originally published.

The committee is indebted to the following administrative units for pictures, materials, and suggestions for the bulletin: *Counties*—Ashe, Beaufort, Buncombe, Carteret, Chatham, Cleveland, Craven, Duplin, Durham, Edgecombe, Forsyth, Greene, Halifax, Harnett, Hoke, Hyde, Iredell, Jackson, Jones, Lenoir, Macon, Martin, Nash, Northampton, Perquimans, Robeson,utherford, Washington, Watauga, Wayne; *Cities*—Albemarle, Ashboro, Asheville, Boone, Clinton, Durham, Elkin, Fayetteville, High Point, Lincolnton, Rocky Mount, Shelby, Statesville, Winston-Salem.

We wish to express our thanks to the many teachers who attended the mathematics conferences and helped make decisions with reference to revisions.

Special recognition is given to Henry A. Shannon, Adviser in Science and Mathematics, for the statement on Mathematics and Natural Science and work on the bibliography for the secondary school; and to L. H. Jobe, Director of Publications, for his helpful suggestions about the arrangement and his careful editorial work.

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Arithmetic In the Elementary School

POINT OF VIEW

The curriculum should provide activities from the experiences of children which will promote understanding of the fundamental processes and principles of arithmetic. Concepts should be developed through the use of concrete materials. Abstract quantitative situations should be introduced gradually. Attention should be centered upon relationships and principles of operation. Skills should be used to solve problems of everyday living.

Mastery of any learning is not acquired all at once; therefore, activities should be planned carefully, so that children will move toward mastery at the rates possible to them. Drills for mastery should be postponed until understanding has been thoroughly established.

It is natural for children to work at different levels of accomplishment and different rates of speed. For this reason, the arithmetic curriculum should be flexible enough to allow for many kinds of differentiation both as to the rates of learning and as to the content. The differences in needs and interests, as well as the differences in abilities, should be recognized and provisions should be made to take care of them.

Arithmetic should be integrated with the entire curriculum. Consideration should be given to the many opportunities to teach arithmetic which arise in all classrooms and in all communities.

SUGGESTIONS FOR HELPING CHILDREN LEARN ARITHMETIC

In teaching children arithmetic there is a great need for attention to be given to the development of meaning. Much of the lack of understanding of arithmetic and the dislike for the subject stem from school programs that stress memorization and abstract drill. Children must see that arithmetic functions in their daily lives. They must see that it is a subject which contains information that they need in order to do many of the things which they want to do.

Little children must do a great amount of work with concrete objects, then semi-concrete, before they work with abstract numbers. No abstract number, no arithmetical term, no process should be introduced to children without sufficient background of activities of concrete nature to insure meaning. Children of all ages need to

Development of Meaning

work with concrete materials and with actual life situations within their experiences in order to discover meanings. Schools should assemble concrete materials that can be used by classes to add meaning to their work in arithmetic.

Some other suggestions of ways to develop meaning:

1. Teach 10 as a basic group. The *teens* will give more trouble than the 20's or 30's. Have each child put 10 sticks in a pile and put a band around the pile. Construct 11 by adding 1 stick to the pile. In like manner work through all the *teens*. When 20 is reached there will be 2 *tens*. Carry as far as necessary to build understanding. Prove that two tens and 2 are 22 by taking the bands off the two bundles of 10's and counting the 10's and the 2. Prove many of the numbers in the same way. Let the children know that *teen* means *and ten*. For example, *thirteen* means three *and ten*. Eleven was once one-teen and twelve was two-teen.
2. Stress place value. A simple chart marked off into places for 1's, 10's, and 100's will help in the beginning of the work. Continue to stress place value in all of the fundamental operations when the children are ready. Work, similar to the following will help:

Addition

Read the following numbers and let the children figure out how they should be placed in a column: seven, one hundred, twelve, two, one hundred six.

Subtraction

237 Borrow 10 from 30, add it to 7, making 17. Subtract —118 8 from 17, etc. It is very misleading and inaccurate to borrow 1 from 3 to add to 7 to make 17. It just is not true.

Multiplication

23 $2 \times 3 = 6$. Since 6 is a unit, put it in the units' place.
 $\times 12$ $2 \times 2 \text{ tens} = 4 \text{ tens}$. Put 4 tens in the 10's place. In 12
 — the 1 is ten. $1 \text{ ten} \times 3 = 3 \text{ tens}$. Put the 3 tens in the
 46 10's place. $1 \text{ ten} \times 2 \text{ tens} = 2 \text{ hundreds}$ ($10 \times 20 =$
 23 200). Put the 2 hundreds in the 100's place. Show
 — that the 23 tens equal 230. Avoid disregarding any
 step because of its simplicity.

Division

$$\begin{array}{r} 20 \quad 16 \text{ tens} \div 8 = 2 \text{ tens. Place 2 tens in the 10's place. } 8 \times 2 \\ 8 \overline{)160} \text{ tens} = 16 \text{ tens. Show that 16 tens equal 160. Show that} \\ \underline{16} \text{ zero is the place holder.} \\ 0 \end{array}$$

3. Stress the fact that zero is a place holder. Work many examples with the children to show that zero's only function is to hold the place.
4. Work with measures. Collect standard measuring vessels. Let the children measure liquids until they understand that two pints make a quart, etc. Let them work in a similar way with other liquid measures and with dry measures. Cut a linear inch, foot, and yard from construction paper or tag-board. Let the children estimate, compare, and measure lengths. Stress the importance of measuring accurately. Work in a similar way with square and cubic measures.
5. Encourage the children to observe the geometric forms around them. Classrooms are filled with rectangles, angles, etc.

Work with concrete materials simplifies tables of measurements



6. Avoid letting arithmetic become an isolated subject. There should be a close tie-up between arithmetic and everyday living.

The wide range of abilities within any class, the difference in readiness and interests, the lack of success, and the danger of maladjustment when mass teaching is used make it necessary to provide for individual differences. A study of the children through observations, tests, analysis of cumulative records, and interviews will disclose the differences and the needs. The needs should be met through a teaching procedure that will promote growth in each individual. Some ways of providing for individual differences are: grouping children within the class according to their needs, keeping the grouping flexible in order to avoid

Provision for Individual Differences stigma; beginning where the children are realizing that all members of the class will not be at the same place; providing materials on many ability levels; enriching the program so that the work will be a challenge to the fast learners; broadening the scope of the work so that the slow learners can achieve success; and using as much individual instruction as possible. Units of experience that emphasize the social and economic phases of arithmetic can be used to a great advantage in bringing all members of the class together for some periods with each member working on his own level of ability and making some contribution of value to the class. Remedial work should be in the light of individual needs. It should consist of remedial instruction as well as remedial drill.

Some of the major causes of children's difficulty in solving problems are: unfamiliar terminology, inability to read understandingly, inability to think through problems, lack of knowledge of the fundamental

Problem Solving operations, unfamiliar settings of problems, and use of careless procedures. From the beginning of a child's school life, he should be guided in building up his vocabulary of arithmetical terms. As early as is consistent with understanding, he should be guided away from using simplified terms, such as *take away* for *subtract*. Lack of ability to read arithmetic problems understandingly is tied up often with the carry-over of simplified terms beyond the point of necessity. Early in the child's school life consideration must be given to training him to think through problems. This training should begin the first year of school, using simple problems from everyday experiences, such as "How

many chairs do we need for Jim's reading group?" The problems in the first year should be quite simple and should grow out of school experiences. The work should be oral.

When children begin to work with written problems, a definite plan for solving problems should be started. The following steps should be included in the plan: Read the problem carefully; decide what should be found; see what facts are given; decide what processes should be used; see if the answer found answers the question the problem asked; see if the answer is reasonable; see if the work checks. In analyzing problems, children must see clearly what they want to find, what facts are given, and how to use these facts to arrive at a solution. Even then they are seriously handicapped in problem solving if they do not have a thorough command of the fundamental operations, since these are the tools for carrying out the solutions. In order to help children solve problems, it is necessary to follow their line of reasoning. It is more important to get children to tell why they worked problems in a certain way than it is to get them to tell how they worked them. To counteract the tendency of many children to use the trial and error method, exercises in solving problems without numbers should be used. Such work will center attention on principles. Frequently children can be helped in problem solving by working orally with simple problems involving the same principles as the ones which are causing trouble, using many problems that grow out of their experiences so that settings will be familiar, and by analyzing problems with the group. Children have less difficulty with problems if they feel the need for solving them; therefore, the best problems are those that are tied with home, school, and community in the local situation.

Drill has two purposes: (1) to make responses to given stimuli automatic and (2) to increase retention. Therefore, it is very important that drill should follow—never precede—thorough un-

Drill derstanding of the ideas involved, and that drill periods should be frequent. If understanding is not built before drill is undertaken, there is great danger of children practicing their errors, developing resistive attitudes, and memorizing instead of reasoning. Drill periods should be short and interesting. Accuracy should not be sacrificed to speed. Each child should be encouraged to keep a record of his own drill accomplishments in order to add interest and to build a desirable form of competition—that is, competition

against one's own record. The spirit of play may be used to bring dull periods to life.

If a skill is to be maintained, drill on it cannot be entirely abandoned. Efficient drill should begin on a skill after understanding has been established and should continue throughout the school years on the level of each child's ability. Special attention should be given to drill in the four fundamental operations with whole numbers, mixed numbers, and common and decimal fractions. Anything that gives meaning to that which is being learned aids in its retention. Helping children to generalize and to see that certain principles always hold true, helping them to see relationships, and encouraging them to use the skills they have learned to solve problems in their own experiences—these things will help them to maintain arithmetic skills.

With the aim of making arithmetic meaningful to children, concrete materials are used to build arithmetical concepts; arithmetic is tied closely to everyday living in school, home, and community; emphasis is placed on the social and economic phases of arithmetic. The fact that many ability levels are found in every classroom is recognized, and instruction and materials are provided to meet the individual needs. Children are grouped within the class for work, and the grouping is kept flexible. The mental maturity of the children is taken into consideration in determining the placement of the work program. Teachers watch

Current Practices in Teaching Arithmetic

A daily chore is turned into a learning situation



for undesirable practices which some children use in their arithmetic work; they help children to substitute desirable practices for undesirable ones. A few undesirable practices which some children use are:

1. Using inaccuracies, such as $4\frac{1}{2} = 2/4$ for $4\frac{1}{2} = 4\ 2/4$.
2. Using careless substitutions, such as *dot* for *decimal*.
3. Memorizing facts without understanding them.
4. Continuing to use simplified terms instead of building a vocabulary of arithmetical terms.
5. Juggling the numbers in problems in an effort to find answers instead of reading the problems carefully.
6. Holding on to crutches after they have served their purpose.
7. Using incorrect language forms.
8. Wasting time while one child struggles at the blackboard with a problem.
9. Working carelessly.
10. Regarding disabilities in arithmetic as permanent.

In planning homework for children, teachers should consider such things as the following: the children's growth, the time children need for family living, rest, recreation, and home duties;

Homework the length of time some children spend on buses; and the lack of equipment for study in some homes. The requirements for homework should be flexible in order to meet individual needs. Individual assignments are desirable. Much of the homework should be of the enrichment type; it should encourage independent study; it should provide opportunities for children to explore their interests. Many of the activities may be self-chosen; many may be creative. Home and community resources should be utilized. If homework is of the drill type, care should be taken to see that the children know what to do and how to do it. A new process should never be assigned as homework.

Games and devices can be very helpful in arithmetic work if care is taken in their selection and use. Some games do not contribute enough to the children's learning of arithmetic to justify

Games and Devices their use; some are set up in such a way that a few children do the learning while the others waste time. A thorough understanding of the processes involved should be established before drill games are played. The same holds true with the use of drill cards. Information about games and devices can be found in many of the books on

teaching arithmetic. Some of the following games and devices can be adapted to several ability levels.

1. *Pictures to Talk About*

Select colorful, interesting pictures that are large enough for the children to see easily. Paste the pictures on cards made of tag board. Through questions about the pictures build understanding of *first*, *last*, *tallest*, *over*, etc.

2. *Number Cards for Beginners*

Make the cards as large as necessary in order to arrange pictures on them. On the first card, paste the picture of one thing of interest to children. Put the number symbol 1 and the word *one* under the picture. Continue in the same way through 10.

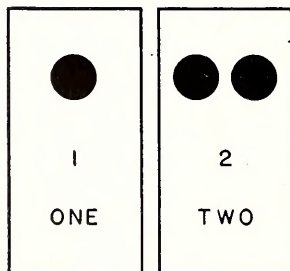
3. *Number Picture Books*

Make a number picture book with the title *All About Four*. Select pictures that show four things, such as four children or four dogs, and paste in the book with the number 4 and the word *four* printed or written in manuscript under each picture. After a sufficient number of pictures that show *four* have been put into the book, select other pictures that show combinations that make *four* and paste in the book. Make other books about other numbers. The number picture books make interesting material for reading tables.

4. *Materials for Counting*

Cut circles and squares from colored tag board. Make them large enough for beginners to handle easily. Use them for counting and grouping. Collect and sterilize ice cream sticks. Use them for counting and working with tens. Flat objects are better for counting than round ones.

5. *Number Dictionary*



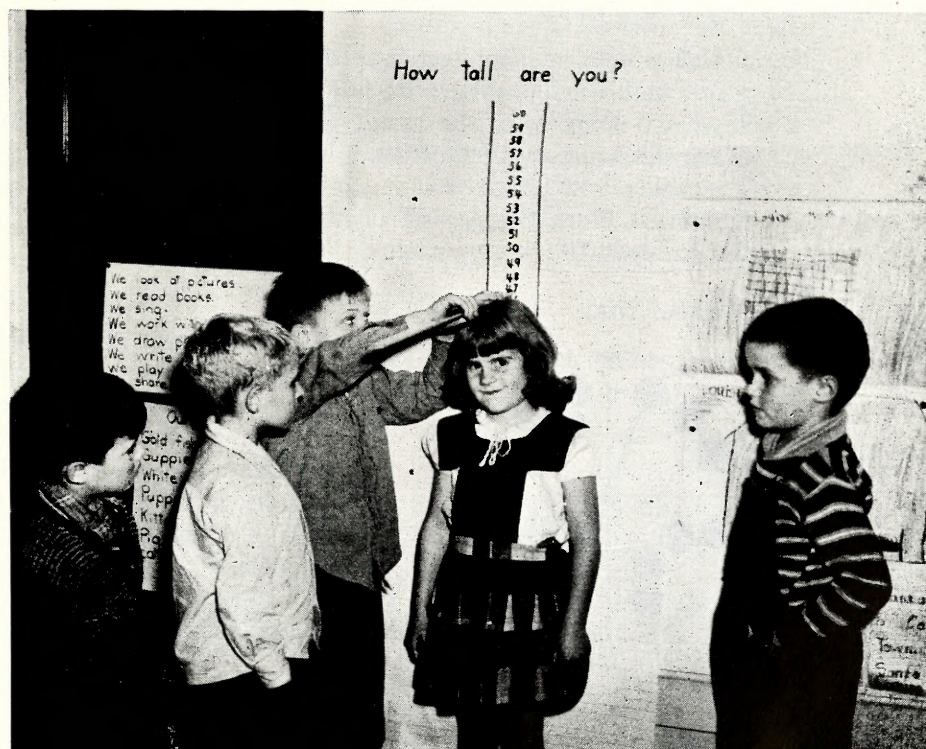
Make the number dictionary through *ten*. The information can be put in a scrapbook (number dictionary), on cards, or on charts.

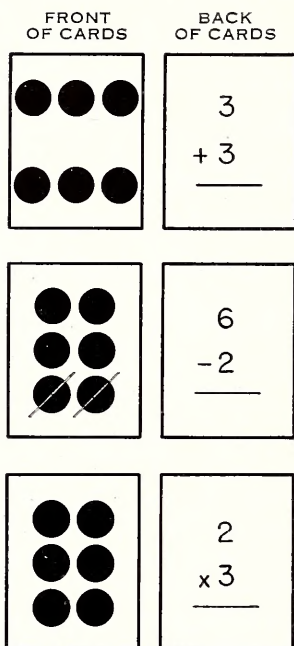
6. Number Puzzle

1		3
	5	
7	8	

Turn some of the numbers on the cards toward the children; turn some of them away. Let the children name the missing numbers. The cards can be drawn on the blackboard and the children can fill in the blank spaces.

Many arithmetic concepts are gained from a simple measuring device

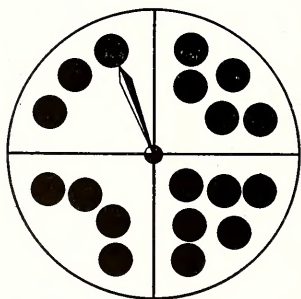


7. *Number Cards*

Make the cards about 8" x 5" if they are to be used in group work. Emphasize the grouping of semi-concrete objects. At the beginning of the work with the cards, the fronts should be used more than the backs. After the concepts have been established, the backs should be used more; the fronts should be used then to verify the answers. Much independent work for children can be built on these cards.

8. *Measuring Device*

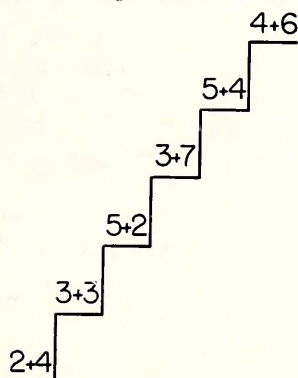
Cut a piece of heavy cardboard about six inches wide and a few inches longer than the tallest child in the room. Use a yardstick to make the inches and feet on the cardboard. Mark the inches with short lines and the feet with long lines. Use color and pictures to make the measuring device attractive. Place the device where it will be convenient for the children to use to measure themselves..

9. *Number Disk*

Cut an eight-inch disk from heavy cardboard. Fasten a marker to the center with a nail. Put a cork on top of the nail to prevent accidents. Have the marker loose enough to spin easily. Collect and sterilize ice cream sticks to use in counting cores. Let each child spin the marker and take as many sticks as the

number of balls in the section in which the marker stopped. At the end of the game, let each child count his sticks to determine his score. For second and third year pupils, numbers can be used and each child can add his score.

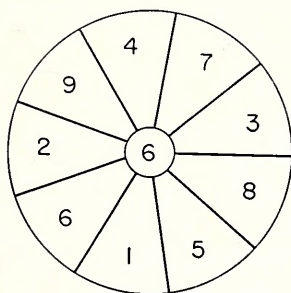
10. *Climbing Stairs*



Draw the stairs on the blackboard. Let each child begin at the first step and climb as far as he can by giving the correct answer to the combination on each step. The child who reaches the top may become the teacher and may choose his pupil from the group. If his pupil reaches the top, he becomes the pupil-teacher. This game can be used for subtraction and multiplication.

Climbing the Ladder and Stepping Stones are similar games.

11. *Number Wheel*

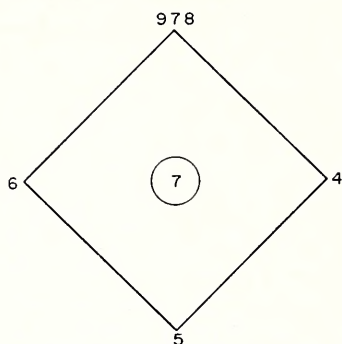


Draw a number wheel on the blackboard. Write the digits in random order near the rim of the wheel. Place the multiplier in the center. Let the children, one at a time, give the products, moving around the wheel in the order that the digits are given. This game can be used for addition combinations.

12. *Magic Square*

	8	4	
	5		
7	2		3
	3	7	6

Fill in each empty square so that the sum of each row, vertically and horizontally, will be the same. The children may be interested in making magic squares for members of the group.

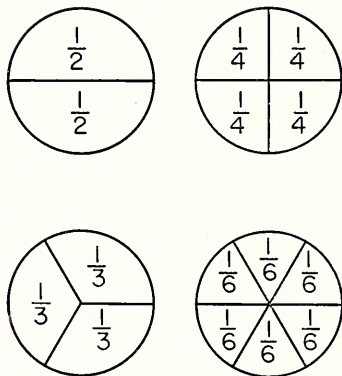
13. *Number Baseball*

Draw a baseball diamond on the blackboard. Write three numbers at the homeplate so that the batter can have three strikes if necessary. Write one number at each of the other bases, and write the multiplier in the pitcher's box. Divide the group of children into two teams. Use baseball rules concerning outs, innings, etc.

Count a score for each player who gives the product of the multiplier in the pitcher's box and the multiplicand at each base. The game can be speeded up by allowing only one strike.

14. *Filmstrip of Steps in Long Division*

Make a drawing of filmstrip on the blackboard. On the filmstrip write the explanation of what was done in each step. Leave the drawing and explanation on the blackboard until the children become familiar with the steps. If the blackboard space is needed, the filmstrip can be drawn on tag board.

15. *Fraction Concepts*

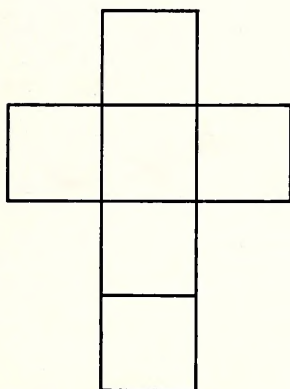
Make two of each figure; cut one into fractional parts; leave the other intact. Let the children place the fractional parts that have been cut upon the figure left intact to prove that $2/4 = 1/2$, $2/6 = 1/3$, etc. Carry as far as needed. Measure accurately.

Make a chart showing many kinds of figures marked into fractional parts.

16. *Measurements*

On the blackboard draw heavy lines of different lengths—12", 18", 9", etc. Draw some of the lines vertically and some horizontally. Measure accurately. From colored construc-

tion paper cut strips the same lengths as the lines on the blackboard. While in his seat, let each child select a strip which he thinks is the same length as that of a designated line on the blackboard. Let the children measure the line with the strips they have selected. Work of this type will help children learn to estimate lengths and widths with a satisfactory degree of accuracy.



Cut a linear inch, a linear foot, and a linear yard from colored construction paper. Mount on tag board and exhibit in the classroom. This should be the work of the children.

Make each block a square inch and fold along lines to make a cubic inch. Follow the same plan to make a cubic foot, making each block a square foot. Exhibit in the classroom. This work should be done by the children.

ILLUSTRATIONS OF HOW CHILDREN HAVE USED ARITHMETIC

Many schools in North Carolina have reported activities that show how boys and girls have used arithmetic. The following activities which have been carried on successfully in many classrooms may suggest some ways in which arithmetic experiences can be brought naturally and meaningfully into the school work of others. A first year group reported on a Christmas party:

This is December 14, 1949.
 We went to town yesterday.
 We bought things for our party
 The napkins cost 15 cents.
 The cups cost 90 cents.
 The cookies cost 40 cents.
 We spent \$1.45.
 We still have \$1.55.

A second year group reported:

"Numbers are fun! We know, because every day we have fun buying and selling in our Corner Grocery Store. We made this store ourselves in the corner of our classroom. . . . We visited a nice, new grocery store. The next day we started our own

store. Mac, David, and Joe made the shelves. . . . The shelves measured 23 inches long and 7 inches wide. . . . Brenda brought her cash register, adding machine, and scales. She showed us how to use them. Mary brought her telephone. We learned how to dial. . . ."

A child in a combination primary group reported:

"The first thing that we do every morning is to count the cafeteria money. On Mondays we take the dollars which pay for lunches all week. We collect twenty cents from the children who pay each day. We keep the record on the blackboard. On Friday we add the money collected during the week."

A P.T.A. committee chairman reported:

"We have a school store that sells school supplies, and it is managed entirely by our third grade children. . . . One child writes on the board the amount of money collected for each item sold during the selling period, and the class totals the sales. . . . The use of zero is understood. . . . The use of the decimal point comes in naturally. . . . The difficult process of carrying and borrowing becomes meaningful."

Teachers reported:

"Beginners learned from the construction of a play house the meaning of many quantitative words."

"There are situations arising many times each day which present opportunities for teaching arithmetic. For instance, the children and I work together in checking the attendance each morning."

"We had a valentine shop in our room. . . . The children had practice in measuring, using money terms and signs, adding, subtracting, and multiplying."

"The refreshment committee planned to make candy for the valentine party. The class figured out the amount of candy that would be needed for the party. It was necessary to increase the measures in the recipe. This called for multiplication and work with fractions. Finding the cost of the party called for much work with fundamental processes."

"Before Christmas the class had a doll shop. Cowboy dolls, clowns, ringmasters, and teddy bears made the shop interesting to the boys as well as to the girls. In putting price tags on the dolls, the children learned to write the dollar sign and the cent sign. After Christmas they had a January clearance sale. All dolls were reduced $1/2$, $1/3$, or $1/5$."

"The children brought familiar vessels to school: milk bottles (half-pint, pint, and quart), fruit jars (pint, quart, half-gallon, and gallon), a quart can, a gallon bucket, and a peck measure. Using water that had been colored with cake coloring to

make it easily seen, the children proved the information given in the table of liquid measures. Using cracked grain, they did the same thing with dry measures. When they had finished the concrete work, one boy remarked, 'Well, seeing is believing'."

"The children measured the windows of their classroom to determine the amount of material needed for drapes. They figured the cost of the drapes; then they figured the total amount of other class expenses to see if they could afford to buy the drapes."

"Each child weighed and measured himself and recorded his weight and height. He compared his weight and height with his former record, subtracting to find out how much he had grown."

"The fourth grade had an arithmetic contest for the purpose of creating interest and motivating drill work. Two captains were chosen, and they, in turn, chose their teams. . . . The daily scores were kept on a chart, and the winning team was the one that accumulated the greater number of points throughout the contest period. . . . The contest developed interest in arithmetic and an eagerness to learn, as each pupil wanted to be prepared to earn points for his team. . . . The children helped each other during the study and practice periods, thus a spirit of helpfulness and cooperation was developed."

"Several of the fourth grade children had bought clothes during sales. . . . Sales were discussed. . . . Each pupil brought his problems to the class to be solved." (Problems were based on local sales.)

"The children used geometric designs to make church windows for a Christmas pageant."

"Each day the children would mark through the date on the calendar with a crayon and then count to see how many days before our party."

"The children came back from the Christmas holidays, walked into the room, and as usual announced something, 'This is the New Year. It's 1950.' I told them that a New Year is important and that this New Year is an especially important one. This year is a new half century. We talked about a century—that it is a hundred years—and that a half century is fifty years. . . . The calendar was used to look at 1950, to learn about the first month of the year, the first day of the month, the number of months in a year, the number of days in the months. Discussion went back to the century and half century. I wrote 1950 on the board again—50 years gone by in this century. . . . Half a century is half of a hundred years—50 years—the new year is 1950, the beginning of another half century. . . . A child suggested that we make a mural for the bulletin board about the things that had happened in the last half century. . . . The group talked about the events and decided to name the mural 'Events of the First Half

Century'. Each child who had volunteered to draw a part of the mural took his turn while small groups worked with finding dates on the calendar and subtracting years to find out how many years between what happened then and now. The children wrote dates—the month, day, year—using the comma to separate day and year. They learned that the New Year is always 1 week—7 days—after Christmas. No, it didn't take long. The morning's discussion furnished creative work for the children. During the day, while I was working with small groups in the regular daily pattern, children by twos and threes looked for information in an encyclopedia, drew pictures, discussed the pictures, and planned for arrangement and spacing of the pictures and for the lettering of the mural. These children had a great thrill over the mural. They called it history and felt that they were doing important work in learning about happenings of the past. . . . There's a difference in helping children to create and in giving them busy work."

Other activities reported which involve arithmetic work were:

1. Making graphs to show school attendance.
2. Working with classroom measurements for many practical purposes.
3. Finding the cost of the supplies furnished by the school; finding the total cost and the cost per pupil.
4. Reading the school electric meter and finding the cost of the electricity the school used.
5. Operating a school bank.
6. Making geometric designs and figures.
7. Reading thermometers and keeping records of the weather.
8. Planning the arrangement of materials on bulletin boards.
9. Planning trips.
10. Planning and issuing a school newspaper.
11. Making arithmetic exhibits.
12. Electing club officers.
13. Planning and making bird houses and feeding stations.
14. Making and using calendars.
15. Making Christmas cards and gifts.
16. Making budgets.
17. Finding the cost of school lunches.
18. Ordering from catalogues.



Seventh year pupils put their finishing touches to their exhibit of geometric designs and figures

19. Decreasing and increasing measures in recipes.
20. Keeping scores in games.
21. Figuring batting averages.
22. Making maps of the classrooms, school grounds, and communities. Drawing to scale.
23. Making school gardens.
24. Planning patterns for weaving.
25. Planning an orderly arrangement of the classroom.
26. Making stages for puppet and marionette shows.
27. Keeping household accounts.

INSTRUCTIONAL MATERIALS

An abundance of concrete materials should be provided for instruction in arithmetic in order to establish meaning and to build arithmetical concepts. Many of the materials in the following lists can be made; some can be collected from homes and business places; some are parts of school equipment. It is recommended that teachers add to the following materials as needs arise.

General:

Library books
 Supplementary books
 Yardsticks
 Tape measures
 Rulers
 Scissors
 Paper (construction,
 drawing)
 Crayons, paints
 Scales

Devices for measuring height
 Pictures with implications of
 arithmetic
 Charts
 Blackboards
 Bulletin boards
 Catalogues
 Thermometers
 Liquid measures
 Dry measures
 Tests

Primary Years:

(Use materials listed under *General*)

Blocks
 Sticks for counting and assembling into tens
 Squares and circles for counting and assembling into groups
 Spools
 Boxes for materials for counting
 Calendars with large numbers
 Rulers with inch and half-inch markings

Number charts
 Number cards
 Large thermometers
 Puzzles
 Toy dial telephones
 Tickets
 Clock faces
 Paper plates
 Textbooks for third year
 Number frames

Children learn to tell time by working with clock faces they have made from paper plates



Middle Years:

(Use materials listed under <i>General</i>)	Blocks and circles cut into fractional parts
Reference books	Fraction charts
Globes	Place value pockets
Maps of countries	Tag board cut into linear inches, feet, yards
Road maps	
Weather maps	Square inch blocks
Measuring instruments	Square foot blocks
Business forms (checks, deposit slips, money orders, etc.)	Cubic inch blocks
Graph paper	Cubic foot blocks
Graphs (picture, line, bar, circle)	Newspapers
	Textbooks

Upper Years:

(Use materials listed under <i>General and Middle Years</i>)	Time tables
Protractors	Standard time charts
Compasses	Geometric figures
Board feet (sawed from lumber)	House plans
	Government bulletins

Films and Filmstrips:

The Division of Instructional Service, State Department of Public Instruction, has prepared a list of films and filmstrips in the field of arithmetic. This list will be sent to schools on request. It is recommended that school faculties preview films and filmstrips before buying them in order to be sure that they meet the needs of the school. Some films and filmstrips will be helpful in workshops for teachers, since from them teachers will get ideas of ways to work with children.

Many of the professional books have chapters on using instructional materials. Some of these are: Chapter VIII, "The Enrichment of the Arithmetic Course: Utilizing Supplementary Materials and Devices," *Arithmetic in General Education*; Chapter 1, Part III, "Arithmetic Comes to Life," *Audio-Visual Methods in Teaching*; Chapter XIII, "Materials of Instruction in Arithmetic", *How to Make Arithmetic Meaningful*; Chapter 1, "Developing an Understanding of Number", *Teaching Arithmetic in the Elementary School*, Volume I. See bibliography for full reference list.

SUGGESTED SEQUENCE IN DEVELOPING FUNDAMENTAL CONCEPTS, PRINCIPLES, AND SKILLS IN ARITHMETIC

The sequence outlined by school years in the following pages was planned for children who make average progress in the learning of arithmetic. Not all children make the same progress; therefore, it is necessary to adjust the arithmetic program to fit the children. It is necessary to find where each child is in his arithmetic learning and to begin there.

It is recommended that a diagnosis be made of the children's arithmetic work at the beginning of each school year in order to discover strengths and weaknesses. The use of new experience will make the review interesting and helpful. Often during the year it may be necessary to review and reteach.

It is recommended that children be trained to check their work for accuracy and neatness as one means of establishing confidence, developing habits of carefulness, emphasizing the importance of accuracy and giving them a basis for evaluating their work.

It is also recommended that the arithmetic program be enriched rather than accelerated in order to avoid plunging children into activities beyond their range of comprehension.

FIRST YEAR

Children enter the first year of school with varying degrees of number readiness. Through the use of concrete materials and classroom situations, number readiness can be built and number concepts can be developed. The arithmetic program in the first year should not be left to chance. It is necessary to make definite plans for the work.

Many situations in which quantitative relationships occupy prominent places may be used for stimulating the learning of needed skills. For example, a pet may be kept in some classrooms. How to get the pet, how to build a pen or cage, how to provide food, when to feed the pet are a few of the questions which will arise. Many activities which call for work with numbers will grow out of such classroom situations.



Anticipation adds meaning to measures

Vocabulary.

1. Use such words as the following:

Words denoting position:

first left before top high over up inside
last right after bottom low under down outside

Words denoting size:

big long wide tall short narrow small
biggest longest widest tallest shortest narrowest smallest

Words relating to measures:

Time: day, night, morning, afternoon, days of week, months, year, yesterday, today, tomorrow, hour—as 12 o'clock noon, 8 o'clock bedtime, but not necessarily to know time on the clockface.

Weight: pound. The child should have some ideas of his own weight, as 40 pounds, 45 pounds, etc.

Length: inch, foot, yard. While the child will learn to talk about these linear measures and know that a foot is less than a yard, he will not be expected to do much measuring.

Liquid measure: pint, quart, cup. These are dealt with daily. The child should have an idea of which is the smallest, next smallest, etc.

Form: line, square, circle (recognition of).

Quantity: whole object, part of object. Begin the fraction concept. Develop concept of half through dividing into groups for games, dividing materials, etc. Use no written symbol.

Money: cent (penny), nickel, dime, quarter.

2. Extend the vocabulary as needs arise.

Number Concepts.

1. Strengthen the serial idea of numbers.

Count by 1's to 10. After a thorough understanding of counting by 1's to ten has been established, count by 1's to 50 or 100. Encourage children to count real objects until the connection between *counting* and *counting objects* has been established.

Count by 5's to 100, then by 10's to 100, after concepts of 5 and 10 have been developed.

2. Develop the group idea of numbers.

Recognize patterns or groups of objects to at least 5.

Use small groups in studying the composition of large groups, as 2 sets of 3 objects make 6, or 3 sets of 2 objects make 6. Let the children count to verify.

3. Distinguish between the quantitative value of a number and its ordinal meaning. Example: There are three boys in the group. Bill is the third boy.

4. Read numbers.

Read numbers to approximately 100 by the end of the year. Keep within the limit of the children's experiences and needs.

Verify the understanding of reading numbers by reading page numbers in first year reading books.

5. Write numbers.

Develop the concept of a number and its symbol in a meaningful way before children write the number. For example, show that 2 is really two 1's, but we write the symbol 2 when we mean two 1's.

Write numbers to ten, then 25, and later in the year, to 50.

Develop the correct formation of numbers. When the children begin to learn to write numbers, teach them to begin at the top of each number and to follow the right line of direction. See the handwriting manuals and guides for suggestions.

Addition and Subtraction.

1. Provide many experiences which will make children ready to work with very simple additive and subtractive facts. This work should be with concrete materials. No answers should be memorized.
2. Explore number combinations with sums and minuends to 10. *This work should be entirely exploratory in the first year, and should be done with concrete materials.* It should be taught to mastery in the second year instead of the first.

Problems.

1. Let the problems be incidental and very simple. Let them be from the children's everyday experiences and based on information the children need in order to accomplish something they want to do.

Examples:

How many chairs are needed for Tom's reading group?

How many children are needed to play the story?

Is Susan the tallest girl in the group?

2. Let the work be oral. Occasionally, the writing of numbers can be brought into the work.

Examples:

Write the number that tells how many days before Joan's birthday. (Before a child writes the number on the blackboard, let the group count the days before Joan's birthday, using a calendar that has numbers large enough for all of the children to see.)

Write the number that tells how many girls are here today.

SECOND YEAR

The work of the second year must be carefully built upon that of the first. If the children do not know the work outlined for the first year, they must be taught this through new experiences rather than through rapid review before beginning the work of the second year.



Children build arithmetic concepts along with their houses

The stores which many children make for the second year classrooms furnish meaningful number experiences. Going to business houses to obtain supplies, constructing shelves, making articles for sale, labeling articles with price tags, keeping simple accounts, and using toy telephones and cash registers are only a few of the ideas which will come to mind. Many other classroom situations are equally fruitful for giving meaning to the number skills suggested for this year.

Vocabulary.

1. Continue the use of the first year vocabulary with the addition of these and other terms as needed:

Words denoting position:

above	behind	by	far	
below	beside	near	next	
first	third	fifth	seventh	ninth
second	fourth	sixth	eighth	tenth

Words denoting size:

small	thick	taller	more than
large	thin	shorter	less than

Arithmetical terms:

add	less	whole	how much is left
subtract	figure	equal	how many in all
zero	part	pair	half, halves

2. Develop the idea that we say things in different ways: 2 and 2 are 4, 2 and 2 equal 4, 2 added to 2 equals 4, etc. The concept of equal and equals should be developed with the concept of addition. Care should be taken to use arithmetical terms on the children's level.

Number Concepts.

1. Build recognition of groups of objects when in regular and easily recognizable patterns at least through 9; as 3 sets of 2 objects are 6, 4 sets of 2 objects are 8, and vice versa; 3 sets of 3 objects are 9, 4 objects and 5 objects are 9, etc.
2. Work with 10 as a basic group. Simplify the work by using bundled 10's (10 sticks, 10 tickets, etc.) plus extra 1's.
3. Read and write numbers.

Read and write numbers as far as needed in using books in the second year and in dealing with numbers within the children's experiences. Suggested limits: reading, 200; writing, 100. Develop correct habits of reading and writing numbers.

Addition.

1. Process meaning: Review and extend the idea of addition as the process of combining numbers to get totals. Use concrete work. Emphasize the usual effect of addition; that is, the answer is larger than any addend except when zero is added to a number.
2. Facts: Teach to the point of mastery the basic combinations to 10 as sum. Provide a few *exploratory* experiences with combinations to 18 as sums in order to show that two numbers whose sum is more than 10 can be added. Combinations from 10 to 18 as sums should be taught to mastery in the third year instead of in the second.
3. Computation: Work with column addition one digit wide, three numbers high, sum not to exceed 10; also, two digits wide, two or three numbers high, the sum of each column to be less than 10.

Help children develop concepts of addition through wide experiences with concrete materials. After the first decade combinations have been met repeatedly in concrete situations, number cards may be made and used by the children under the direction of the teacher. The first decade combinations should be

taught under the direction of the teacher. The first decade combinations should be taught to mastery in the second year.

First decade combinations exclusive of zero combinations

1 2 3 4 5 6 7 8 9
1 1 1 1 1 1 1 1 1

2 3 4 5 6 7 8
2 2 2 2 2 2 2

3 4 5 6 7
3 3 3 3 3

4 5 6
4 4 4

5
5

The reverses—addition only

1 1 1 1 1 1 1 1
2 3 4 5 6 7 8 9

2 2 2 2 2 2
3 4 5 6 7 8

3 3 3 3
4 5 6 7

4 4
5 6

Zero combinations

0 1 2 3 4 5 6 7 8 9
0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8 9

The combinations must be developed slowly and carefully, giving children confidence as they proceed from one step to the next. When they have learned what adding 1 to each number means, they can more readily get the concept of adding 2, and later of adding 3, 4, etc. Thorough understanding through 5, using both addition and subtraction, should be gained before working with combinations with sums greater than 5. The addition concept should be developed first through building up before breaking into parts for subtraction.

Children must progress through various stages before they can think in terms of abstract numbers. First they must use concrete materials; later they must deal with semi-concrete materials—pictures of objects, then representations of objects such as dots and dashes. After understanding has been developed, the children should work with abstract numbers. Many second year children need to go through these stages.

The concepts of multiplication and of fractional parts may be easily taught from thorough work with the addition facts. When children know that 2 and 2 are 4, they also know that they used 2 twice to get 4; they know that one of their 2's is only half as much as both of their 2's, and therefore, that 2 is one-half of 4.

Subtraction.

1. Process meaning: Teach the basic meaning of subtraction as the process by which one part of a number is taken away (the how-many-left idea). Relate to the idea of addition.
2. Facts: Teach to the point of mastery the basic combinations to 10 as minuend. Relate addition and subtraction facts. Provide a few *exploratory* experiences with combinations to 18 as minuends in order to show that such numbers can be subtracted. Combinations from 10 to 18 as minuends should be taught to mastery in the third year instead of the second.
3. Computation: Subtract two-figure subtrahends from two-figure minuends without borrowing.

Multiplication.

1. Show how two 2's make 4, two 3's make 6, etc., as well as $2 + 2 = 4$, $3 + 3 = 6$, etc.; also show that three 2's make 6, as well as $2 + 2 + 2 = 6$. The purpose is to develop the concept of multiplication as another, and shorter, way of adding.
2. Develop a thorough understanding of additive facts before introducing multiplication.

Part-Whole Relationships.

1. Extend and enrich the meaning of fractions. Let the idea of fractions as involving parts of wholes emerge from natural situations in the classroom.

2. Develop concepts of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$; but do this slowly. Show how these fractions are parts of groups as well as parts of wholes; as $\frac{1}{2}$ of 4 books, $\frac{1}{2}$ of 1 apple, etc. Use concrete materials.

Common Measures.

1. Time: hours, half-hours, sequence of months and days of the week.
2. Weight: pound, less than a pound, half-pound.
3. Length: inch, foot, yard. Use as units of measures and for comparisons. Provide simple experiences for the use of measures.
4. Liquid measure: pint, quart, more or less than a quart, gallon.
5. Dry measure: peck, bushel.
6. Money: cent, nickel, dime, quarter, half-dollar, dollar.

Problems.

1. Solve one-step problems that arise within the experiences of the children and that are on their ability levels.
2. Encourage children to make problems.

THIRD YEAR

It is important to make the quantitative side of children's experiences come to the front naturally. This can be done in many ways. For example, the children and their teacher may plan a class party. How much food to buy, how many napkins and favors to make, how to find the total cost of the party, and how to pay the cost are a few of the problems the children will need to solve.

Vocabulary.

1. Strengthen the vocabulary with such words as the following:

Arithmetical terms:

prove	fraction	multiply	plus
divide	divisor	product	minus
sum	example	measure	sign
remainder	difference	total	column



A toy rental shop provides many learning situations

Words used in comparing:

large	small	high	low	long	short
larger	smaller	higher	lower	longer	shorter
largest	smallest	highest	lowest	longest	shortest

2. Use the names and symbols for the following: +, —, ×, ÷, =.

Reading and Writing Numbers.

1. Read and write Arabic numbers as far as needed. Stress correct formation and reading of numbers.
2. Read and write Roman numbers through XII.

Addition.

1. Process meaning: Extend and enrich the meaning of the process. (See Second Year.)
2. Facts: Finish all the basic addition combinations to 18 as sum. Work for mastery.

Second decade, 36 combinations

2 3 4 5 6 7 8 9
9 9 9 9 9 9 9 9

3 4 5 6 7 8 9
8 8 8 8 8 8 8

4 5 6 7 8 9
7 7 7 7 7 7 7

5 6 7 8 9
6 6 6 6 6 6 6

6 7 8 9
5 5 5 5 5 5 5

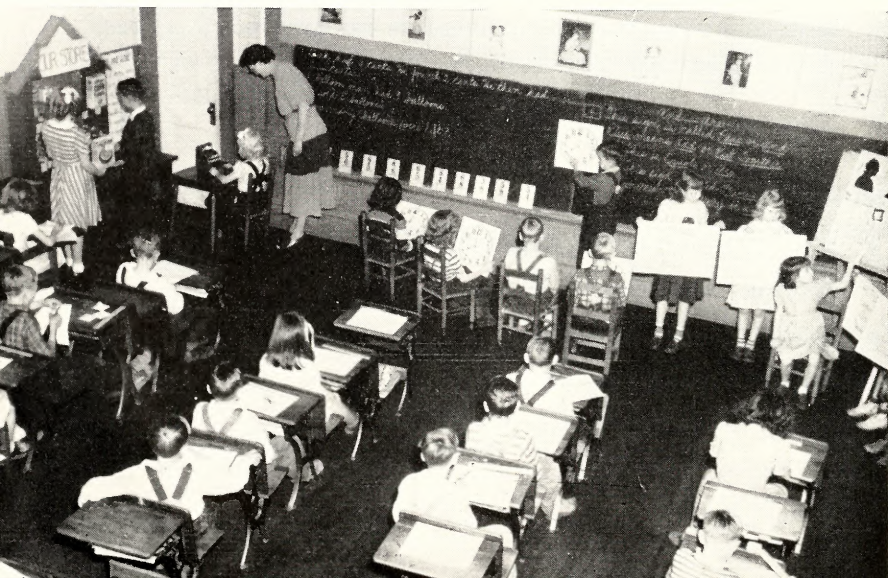
7 8 9
4 4 4 4 4 4 4

8 9
3 3 3 3 3 3 3

9
2 2 2 2 2 2 2

3. Computation: Teach column addition of two to four numbers of one digit each with bridging of decades; also of two or three numbers of two or three digits each without and with carrying.

Despite the lack of modern furniture, many individual and group activities are in progress



Subtraction.

1. Process meaning: Extend the meaning of subtraction to cover ideas other than how-many-left. These ideas are represented in such questions as:
How many are gone?
How much larger is _____ than _____?
How many more are needed?
2. Facts: Finish the study of the basic subtraction facts to 18 as minuend. Assure mastery.
3. Computation: Work with numbers two and three digits wide, first without borrowing, then with borrowing; without zeros, and then with zeros.

Multiplication.

1. Process meaning: Enrich the idea of multiplication as a short way of adding the same number several times.
2. Facts: Teach the multiplication facts through 45, or 5×9 , with reverses. Use concrete illustrations.
3. Computation: Teach multiplication of numbers with two and three figures without carrying and with carrying.

Division.

1. Process meaning: Teach the idea of division as the reverse of multiplication and as successive subtraction of the same number. Example: $15 \div 5 = 3$, because $3 \times 5 = 15$, and $15 - 5 = 10$, $10 - 5 = 5$, and $5 - 5 = 0$. Use concrete illustrations.
2. Facts: Teach the division facts which correspond with the multiplication facts for this year to $45 \div 9 = 5$. Relate the new division and the new multiplication facts.
3. Computation: Teach division of two- and three-figure dividends by a single digit, without carrying and without remainder; long division method to be employed.

Money Concepts.

1. Write numbers using cent signs, dollar marks, and decimal points. Let these numbers come from the children's experiences.
2. Use such numbers in adding, subtracting, multiplying, and dividing by one-digit numbers. These problems should be kept very simple.

Whole-Part Relationships.

1. Extend the concept as need arises.
2. Develop further the fractional concept, particularly in comparisons using only unit fractions, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, and later $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{8}$.

Common Measures.

1. Use understandingly the common measures taught in first and second years. Use actual units of measure.
2. Tell time correctly.
3. Use dozen and one-half dozen.
4. Estimate distances, as yard, foot, inch.

Problems.

1. Solve simple one-step problems that come from the experiences of the children.
2. Keep the problems within the limits of the children's abilities.
3. Encourage each child to check his work for accuracy and neatness.

FOURTH YEAR

The following activity is only one of the many activities that can be used to give significance to the concepts and skills suggested for the fourth year:

The children planned a toy sale to make money for the library. They bought materials and made the toys. They computed the cost of each toy and fixed the sale price. Members of the class arranged the toys for sale, served as clerks, and kept accounts.

Vocabulary.

1. Add such words as the following:

Arithmetical terms:

average	numerator	dividend
division	denominator	multiplier
quotient	minuend	multiplicand

Words used in comparing:

wide	good	much	near
wider	better	more	nearer
widest	best	most	nearest

2. Use abbreviations, such as:

pt.	oz.	ft.	hr.	mo.
qt.	lb.	yd.	wk.	yr.

Reading and Writing Numbers.

1. Read and write Arabic numbers as far as needed.
2. Read and write Roman numbers to XX; use other Roman numbers as needed.
3. Continue to stress the correct reading and writing of numbers.

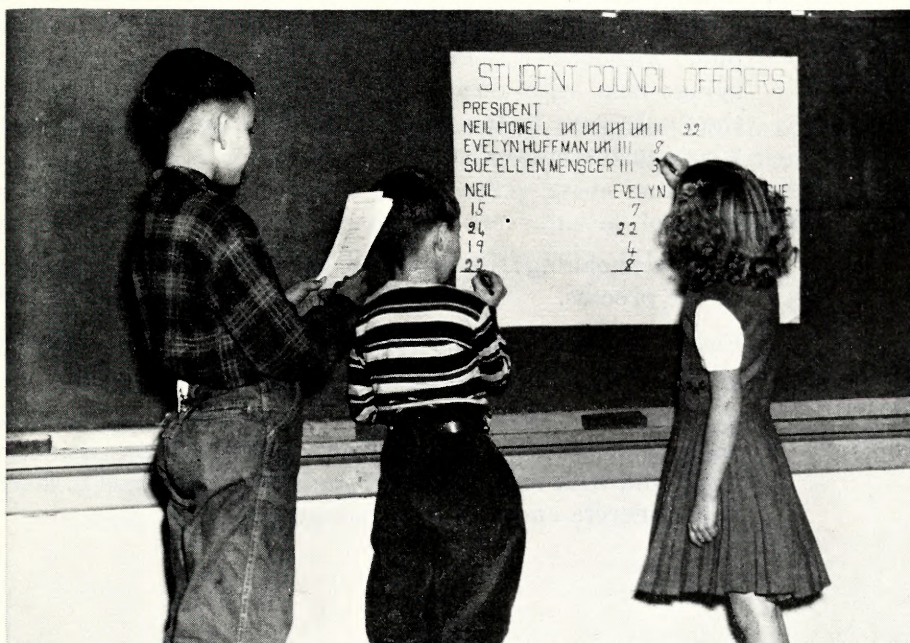
Money Concepts.

1. Strengthen the concepts of comparative values: \$1.00 is 4 times 1 quarter, 10 times 1 dime, 20 times 1 nickel, 100 times 1 penny.
2. Continue the use of money numbers in examples and problems.

Addition.

1. Process meaning: Continue to give attention to the meaning of the process.

The election of officers contributes to the arithmetic program



2. Facts: Practice to maintain mastery.
3. Computation: Work with addition of four- to five-figure numbers in columns of two- to four digits, with and without carrying.

Subtraction.

1. Process meaning: Continue to give attention to the meaning of the process.
2. Facts: Practice to maintain mastery.
3. Computation: Use four- to five-place minuends with two- to five-place subtrahends, with and without borrowing.

Multiplication.

1. Process meaning: Continue to give attention to the meaning of the process.
2. Facts: Review the facts through 5×9 , and master the new facts to 9×9 .

It will be well for the teacher to read the outlines and suggestions for preceding years and to build on knowledges already acquired. The fourth year child probably still needs many experiences with concrete and semi-concrete materials to gain mastery of the multiplication process.

3. Computation: Work with multiplication of two- or three-place multiplicands by a one-, two-, or three-figure multiplier, without zeros, first without carrying, then with carrying; next with zero as a figure in the multiplicand, as the final figure in the multiplier; and last as a figure elsewhere in the multiplier.

Division.

1. Process meaning: Continue to give attention to meaning of the process.
2. Facts: Review the division facts to $45 \div 5$, and master the other simple division facts.
3. Computation: Divide only by one-place divisors; *use the long division form*; use dividends to four places, with carrying and with remainders. Again the child needs to learn with concrete and semi-concrete materials.

Fractions.

1. Extend the meaning of fractions to cover more than one of several equal parts of a whole (denominators to 8, numerators larger than 1).
2. Use concrete and semi-concrete materials in working with wholes and parts of groups. Begin adding like fractions through the use of parts of concrete objects. Help the children to understand that we may add parts as well as wholes: $\frac{1}{4}$ and $\frac{1}{4}$ are $\frac{2}{4}$, $\frac{1}{4}$ and $\frac{2}{4}$ are $\frac{3}{4}$.

Common Measures:

1. Continue the use of measures taught in the second and third years.
2. Use when needed: $\frac{1}{2}$ yard, $\frac{1}{4}$ yard, $\frac{1}{2}$ dozen, $\frac{1}{4}$ dozen, 1 minute, 1 second.

Problems.

1. Use simple one-step problems with and without numbers.
2. Use two-step problems only on rare occasions.
3. Use very simple problem analysis.
4. Encourage each child to check his work for accuracy and neatness.

Story of Numbers.

1. Begin the story of the development of the number system.
2. Provide materials that deal with the story of numbers.

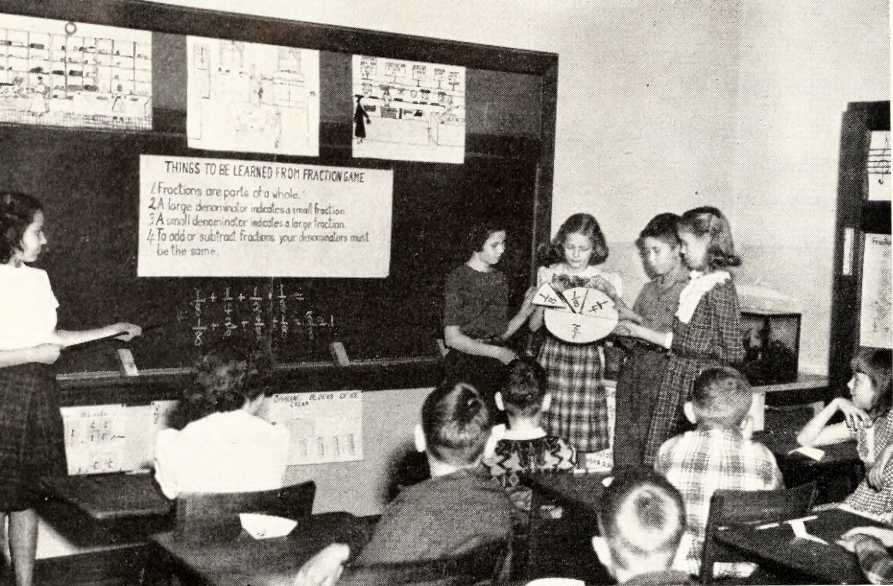
FIFTH YEAR

School gardens provide many practical activities which add to the arithmetic program. Some of these activities are: planning and measuring the gardens, buying seeds and other supplies, keeping accounts, computing the gains or losses. Many other class activities are filled with arithmetic problems which add interest to the program.

Vocabulary.

1. Broaden the vocabulary with such words as the following:

proper fraction	lowest term	invert
improper fraction	decimal	cancel
mixed number	rectangle	graph
common denominator	square	reduce



An understanding of fractions is developed through concrete and semi-concrete materials

2. Substitute the correct terms for simplified terms carried over from preceding years.

Reading and Writing Numbers.

1. Read and write Arabic numbers as far as needed.
2. Read and write Roman numbers to L. Work with others as needed. Use C, D, and M to show how 100, 500, and 1000 are written in Roman numbers.

Fundamental Operations with Whole Numbers.

1. Process meaning: Extend and enrich the meaning of the processes of the four fundamental operations.
2. Facts: Master the basic facts of addition, subtraction, multiplication, and division.
3. Computation:

Addition: Use four- or five-place numbers not more than three to five numbers high.

Subtraction: Use three- to five-place minuends and two- to five-place subtrahends without and with zeros, and without and with borrowing.

Multiplication: Use two- and three-place multiplicands, two- and three-place multipliers, without and with carrying; with zeros.

Division: Use long and short division with and without remainders and with carrying. Use one- and two-place divisors and two- to five-place dividends with and without zeros. Do some work with three-place divisors.

Fundamental Operations with Fractions.

1. Addition and subtraction: Work with fractions with like denominators. Introduce fractions with unlike denominators whose common denominator does not exceed 20.
2. Multiplication: Multiply whole numbers by fractions, such as $4 \times \frac{1}{2}$, $6 \times \frac{1}{3}$, $4 \times \frac{1}{3}$, $6 \times \frac{1}{2}$. Multiply fractions by whole numbers such as $\frac{1}{2} \times 4$, $\frac{1}{4} \times 6$, $\frac{1}{2} \times 3$. Multiply fractions by fractions, such as $\frac{1}{5} \times \frac{1}{5}$, $\frac{2}{3} \times \frac{3}{4}$, $\frac{3}{4} \times \frac{5}{6}$. Develop the idea of why the value of the fraction is decreased when the denominator is increased numerically.
3. Division: Develop the division concept in fractions. Show how the numerator is divided by the denominator. Show how division may be indicated by a common fraction. Example: $8 \div 4$, $\frac{8}{4}$.

Decimals.

1. Extend the use of the decimal system in the calculation of money.
2. Relate decimals to common fractions.

Measurements.

1. Teach recognition of various forms of area: square, triangle, rectangle, etc.
2. Teach understanding and recognition of the practical uses of linear, square, and cubic measures; solve simple problems involving perimeters and square measures.
3. Use the following measure tables as needed: liquid measure, dry measure, time, weight (avoirdupois and troy), linear, square. Develop the learnings through use of concrete materials rather than through memorization.

Problems.

1. Work with one- and two-step problems involving processes already taught.
2. Develop a suitable plan for solving problems.
3. Encourage each child to check his work for accuracy and neatness.

Story of Numbers.

1. Continue the story of the development of the number system.
2. Make the story interesting through the use of many materials.

SIXTH YEAR

There are many class activities that can be used very successfully in the arithmetic program suggested for the sixth year. The activities connected with a class or school newspaper stress business operations; therefore, they can be used to an advantage in the arithmetic work. Some of the activities are: determining the cost of the newspaper, buying the necessary supplies, spacing the materials for publication, keeping accounts.

Vocabulary.

1. Enrich the meaning of the words suggested in the fifth year.
2. Add other words used in the discussion of the work, such as:

budget	percent	scale	dimensions
deposit	account	area	compare
receipt	volume	discount	balance

Fundamental Operations with Whole Numbers.

1. Provide for practice in the four fundamental processes through various activities and projects.
2. Give inventory tests to determine what skills need to be developed.
3. Review and reteach as necessary.
4. Develop home, school, and community problems involving:

Column addition: Use numbers with uneven left hand margins, columns six to eight numbers high composed of numbers of three to six digits.

Subtraction: Use numbers to six places with zeros in both minuend and subtrahend.

Multiplication: Use 3-digit and 4-digit multipliers. Use zero as an internal digit and as final digit in multipliers.

Division: Use two- and three-place divisors with zeros in both the dividend and the divisor.



Doubling and tripling recipes adds interest and meaning to fractions

5. Develop the idea of cancellation as a short process in division.
6. Read, write, and interpret large numbers, such as may be found in newspapers and magazines.

Fundamental Operations with Common Fractions.

1. Addition and subtraction: Work with fractions with like and unlike denominators. Expand the concept of least common denominators. Add and subtract mixed numbers.
2. Multiplication: Multiply fractions by fractions, whole numbers, and mixed numbers. Multiply mixed numbers by mixed numbers. Multiply whole numbers by fractions and mixed numbers.
3. Division: Divide fractions by fractions, mixed numbers, and whole numbers. Divide mixed numbers by mixed numbers, fractions, and whole numbers. Divide whole numbers by fractions and mixed numbers.
4. Use many concrete illustrations of the four fundamental operations.
5. Introduce the principle of cancellation.
6. Reduce fractions to the lowest terms.
7. Work with problems involving fractions.

Decimals.

1. Develop the ability to read and write tenths, hundredths, and thousandths as decimal fractions.
2. Develop the concept that a decimal fraction is a short way of writing a fraction.
3. Change common fractions to decimal fractions and decimal fractions to common fractions, using a variety of illustrations.
4. Use the four fundamental operations with decimals.
5. Work with problems of the home, school, and community involving the use of decimals. (The trend in science, industry, and commerce is towards the use of decimal fractions.)

Measurements.

1. Use concrete materials to aid in the development of the concepts of measures, using standard units of measures. (See *Sixteenth Yearbook* of the National Council of Teachers of Mathematics. pp. 176-181.)
2. Work problems involving the use of denominate numbers; use the tables.
3. Read and make graphs, such as line and bar graphs, using local data.
4. Find perimeters of rectangles and triangles.
5. Find areas of squares, rectangles, and triangles.
6. Find volumes of rectangular solids.

Percentage.

1. Develop the concept of percentage by the use of a variety of concrete illustrations.
2. Emphasize the relationships between common fractions, decimal fractions, and percentage.
3. Find percents of numbers.
4. Work with simple interest.
5. Use problems dealing with money.

Problems.

1. Work with many problems that deal with school, home, and community activities.



An understanding of many business operations is developed through the use of the school bank

2. Enrich problem analysis developed in preceding years.
3. Encourage each child to check his work for accuracy and neatness.

Story of the Number System.

1. Use many illustrations in continuing the story of the development of the number system.
2. Encourage simple research.

SEVENTH YEAR

Activities which will broaden the children's understanding of the economic and social significance of arithmetic should be used. Many of these activities should be based on school, home, and community situations; they should be based on the children's needs and interests. In planning the activities, provisions should be made to take care of the many levels of ability which are found in all classrooms.

Vocabulary.

1. Enrich the meanings of the words used in the sixth year.
2. Add the words used in discussion of the number system, fundamental operations, business practices, measurements, and geometric constructions.

Fundamental Operations with Whole Numbers.

1. Use inventory tests to determine the concepts, principles, and skills that need emphasis.
2. Review and reteach fundamental operations as needed, using problem situations.
3. Practice rounding numbers.

Fundamental Operations with Common and Decimal Fractions.

1. Use concrete illustrations to strengthen concepts of common and decimal fractions.
2. Change decimal fractions to common fractions; change common fractions to decimal fractions.
3. Use the four fundamental operations involving common and decimal fractions. Use in problem situations.

Percentage.

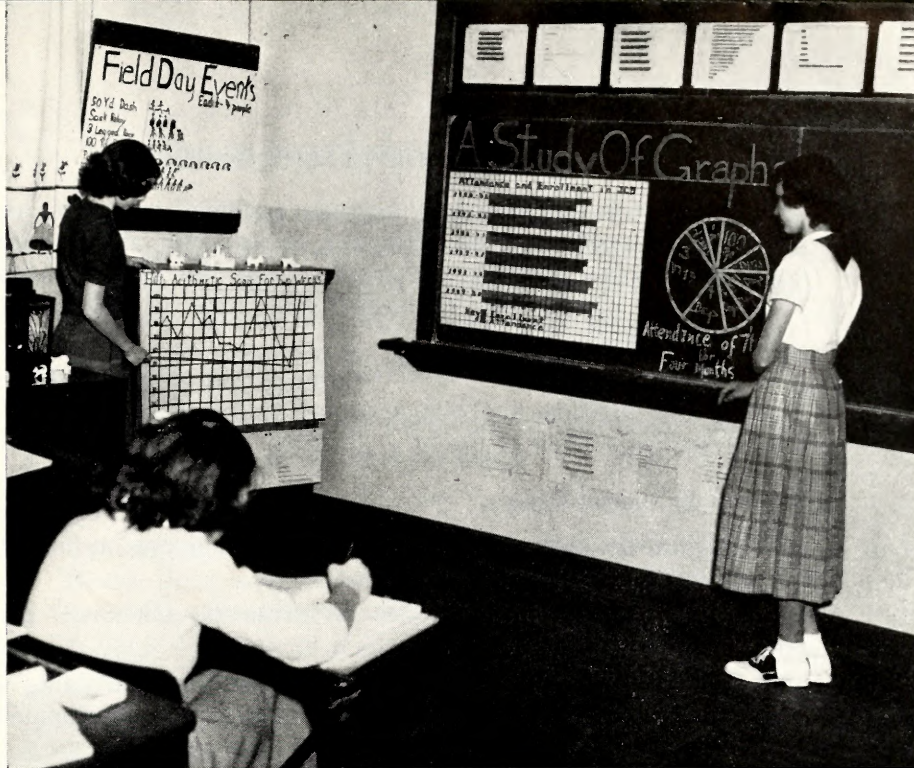
1. Use concrete illustrations to develop the conception of percentage.
2. Establish the relationships between common and decimal fractions and percent. Example: $1/4 = .25 = 25\%$.
3. Develop the concept of ratio with many practical applications.
4. Find percents of numbers.
5. Find the percent one number is of another number.
6. Make use of percentage as follows: discount on bills; commission on sales; percent of increase or decrease in school attendance, population, etc.; simple interest on loans; and problem situations relating to production, athletic records, recipes, etc.

Graphs.

1. Use bar, line, circle, picture, and map graphs.
2. Construct bar and line graphs from data given in the text and from local problems relating to such items as production, population, school attendance, taxes, and wages.
3. Round numbers as related to graphical representation.

Geometric Forms.

1. Introduce the study of simple geometric forms in nature, practical crafts, industry, architecture, etc., for purposes of motivation and orientation.



2. Work with such everyday business problems as buying and selling, profit and loss, discount, commission, simple cases of interest, banking, insurance from the standpoint of sharing risks. Use business forms, such as checks and deposit slips.
3. Use graphs to help to establish the concept of relationships.
4. Enrich problem analysis developed in preceding years.
5. Encourage careful checking of work.

Reports on Special Topics.

1. Encourage each child to contribute to the group on his own level of ability.
2. Make use of reports to help integrate the arithmetic program with other areas.
3. Provide materials for research. Consider such fields as conservation of resources and the story of the development of our number system.

EIGHTH YEAR

In the eighth year many activities should be used that motivate the use of fundamental processes, that stress business operations, and that deal with situations the children meet, and will meet, in everyday life. Provision should be made to take care of the differences in the children's abilities, needs, and interests.

Vocabulary.

1. Enrich the meaning of words used in the seventh year.
2. Add the words used in discussion that have not been developed fully in preceding years.

Fundamental Operations.

1. Test to determine the need for review of the fundamental operations involving whole numbers, common and decimal fractions.
2. Work with the fundamental operations through the year as needed to maintain skills.

Problems.

1. Use problems involving business forms and practices. Consider the following: banking, building and loan, stocks and bonds, insurance, budgeting, simple and compound interest,

relationship between cash and installment buying, taxes, reading time-tables and maps, and computing cost of travel.

2. Use many problems based on the experiences of the children and relating to home, school, and community.
3. Base some problems on current events, safety education, and use of resources.
4. Enrich problem analysis developed in preceding years.
5. Encourage careful checking of work.

Informal Geometry.

1. Use rulers, compasses, and protractors in construction. Bisect lines and angles. Work with perpendiculars and parallels. Inscribe hexagons and other polygons in circles.
2. Develop concepts of symmetry, congruence, similarity, and equivalence, using triangles, polygons, circles, and illustrations from art, nature, crafts, etc.
3. Experiment with congruent and similar triangles. Develop the work informally.
4. Make scale drawings of classrooms, tennis courts, baseball diamonds, etc.
5. Work with the right triangle.
6. Use home-made instruments, such as the transit and the level, for outdoor work.
7. Estimate distances, areas, and volumes; check by actual measurements.
8. Continue the study and use of such formulas as grow out of the measurement of perimeters of triangles, squares, rectangles; areas of triangles, squares, rectangles, trapezoids, circles; surfaces and volumes of prisms, pyramids, cones.
9. Construct and use models of plane and solid figures; use standard units of weights and measures.
10. Determine the relationships between standard units of measure through experiments.

Equations.

1. Use the following techniques to establish equation concepts: make verbal statements and symbolic representation of quantitative relationships, add and subtract similar terms, evaluate equations, solve equations involving not more than two steps.

2. Make and solve problems relating to home, travel, insurance, percentage, etc.
3. Work with ratio and proportion.

Reports on Special Topics.

1. Help each child to contribute to the group on his own level of ability.
2. Use reports to tie school, home, and community more closely together.
3. Provide materials for research.

MEASUREMENT OF PROGRESS

Progress in arithmetic must be measured by more adequate means than just the scores on tests. Dr. Brownell suggests four classes of evaluation techniques: "(1) paper-and-pencil tests (or simply tests), (2) teacher observation, (3) individual interviews and conferences with pupils, and (4) pupil reports, projects, and the like."¹ Dr. Brueckner and Dr. Grossnickle give seven types of evaluation procedures: "(1) standard tests, (2) teacher-made objective examinations, (3) problem situation tests, (4) evaluating something the pupil has produced, (5) behavior records, (6) interest inventories and reports, (7) evaluating the learning process itself."²

Through careful observations much can be learned about children's thought processes and methods of work. Inefficient procedures that may not show up on tests will come to light; so will outcomes that are too intangible to be measured by tests. The development of such outcomes as interests, appreciations, and other desirable attitudes is a part of progress in arithmetic and should be given consideration. Each child should be encouraged to evaluate his own progress. Graphs of individual progress will serve not only as measurements but as incentives for improved work. Such graphs should not be used to compare one child's work with another's.

At the beginning of the school year, teachers should measure arithmetical understandings and computational skills which children have acquired in previous years in order to know where to begin instruction and how to group the children. Standard tests are useful for survey purposes. Teacher-made plans for measurement of progress go hand in hand with good instruction.

¹William A. Brownell. "The Evaluation of Learning in Arithmetic", Ch. X, *Arithmetic in General Education*, Sixteenth Yearbook of The National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1941. p. 235.

²Leo J. Brueckner and Foster E. Grossnickle. *How to Make Arithmetic Meaningful*. Atlanta: The John C. Winston Company, 1947. p. 377.

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- Leaf, Munro. *Arithmetic Can Be Fun*. Philadelphia: J. B. Lippincott Company, 1949. \$1.75. 2-4.
- Leeming, Joseph. *More Fun with Puzzles*. Philadelphia: J. B. Lippincott Company, 1947. 149 pp. \$2.50. 6-8.
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See the list of State adopted supplementary books for reading books that are based on arithmetic.

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- Brueckner, Leo J. and Grossnickle, Foster E. *How to Make Arithmetic Meaningful*. Atlanta: John C. Winston Company, 1947. 513 pp.
- Buckingham, Burdett R. *Elementary Arithmetic. Its Meaning and Practice*. Atlanta: Gun and Company, 1947. 744 pp.
- In the preface of *Elementary Arithmetic. Its Meaning and Practice*, Dr. Buckingham states: "This book is an arithmetic. It is not a book on the philosophy, psychology, or the pedagogy of arithmetic. Its aim is to encourage scholarship in the subject itself. It differs from an arithmetic for children chiefly in the fact that it is addressed to more mature minds."
- Dale, Edgar. "Arithmetic Comes to Life." *Audio-Visual Methods in Teaching*. New York: The Darden Press, 1946. 546 pp.
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PROFESSIONAL MAGAZINES

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- Childhood Education*. Association for Childhood Education International, 1200 Fifteenth Street, N. W., Washington 5, D. C. \$4.50. (Subscription is included in membership dues of \$6.00.)
- Journal of Educational Research*. Dembar Publications, Inc., 114 South Carroll Street, Madison 3, Wisconsin. \$4.00.
- NEA Journal*. National Education Association, 1201 Sixteenth Street, N. W., Washington 6, D. C. (Subscription is included in membership dues.)
- The Elementary School Journal*. The University of Chicago Press, 5750 Ellis Avenue, Chicago 37, Illinois. \$4.50.

Mathematics In the Secondary School

NINTH YEAR

(Course A or B Required)

Course A: General Mathematics.

This course will likely be taken by students who do not intend to pursue mathematics much further. Hence this course should give students a good check-up on what they have studied in mathematics. It should also give them some insight into the more powerful forms of analysis as found in algebra through the use of the equation and the formula, and in geometry through measuring and construction work establishing relationship of areas, volumes, etc.

The following topics are suggested for work in this year; teacher and pupils should select topics and add new topics in keeping with needs and interests.

1. Inventory test on concepts, skills, and principles developed in years VII and VIII. Review and extension of concepts, skills, and principles relating to integers, decimals, fractions, fundamental operations, estimating and checking, denominate numbers, percentage, tables of squares, square roots, compound interest and other tables dealing with money matters.
2. Development and use of formulas for areas, perimeters, and volumes of such figures as pyramids and cones. Take a cone and a cylinder of the same base and altitude, fill the cylinder with sand and then see how many times this will fill the cone. Do the same for a pyramid and a prism.
3. Review of the basic concepts relating to the angle, triangle, and circle, making geometric constructions.
4. Constructions: triangle given sas, asa, sss; triangle similar to a given triangle; find the center of circle when an arc is given; angles of 30, 45, and 60 degrees.
5. Informal proofs of elementary theorems by measurement and construction. Typical exercises:
 - a. The sum of the angles of a triangle equals 180 degrees. Cut out paper or cardboard triangles and cut off angles and place together to form a straight line.

- b. The Pythagorean theorem. Employ usual construction of right triangle with squares on hypotenuse and legs divided into units of equal measure. Multiply both sides of the equation $c^2 = a^2 + b^2$ by π over 4 and interpret the result: The circle on the hypotenuse as diameter is equal to the sum of the two circles on the legs as diameters.
6. Graphs: Draw circle graphs; read and construct more advanced bar and line graphs based on data of significance to pupils; use graphs in study of linear equations.¹
7. Story of the development of geometry including indirect measurement.²
8. Use of home-made instruments for measuring, such as transit level, hypsometer, proportional compasses, adding machine.
9. Story of algebraic symbolism.³
10. Evaluation of formulas. Example: The destructive power of an automobile is given by the formula $K = WV^2$, where W = weight of car and V = speed of car.
 Let $W = 3000$ lbs. $V = 30$ miles per hour
 $V = 60$ miles per hour
 $V = 90$ miles per hour
 Note that in doubling the speed the destructive power is four times as great, and that trebling the speed makes the destructive power of the car nine times as great.
11. Review and extension of concepts of formula, equation, similar terms, ratio and proportion.
 Example: the distance a car will run after the brakes are applied is proportional to the square of the velocity, or as expressed in a formula $D = .07V^2$, where D = distance in feet, and V = miles per hour. Plot this equation on graphical paper.
12. Develop concepts of low powers (a^2 , a^3), positive and negative numbers as found in current magazines.
13. Fundamental skills, techniques and principles for dealing with the equation and the formula:
 - a. Four fundamental operations involving positive and negative numbers, using numerals and letters to represent numbers.

¹See Graphs. **How to Make and Use Them**, by Arkin.

²See **History of Mathematics** by Sanford.

³See **History of Mathematics Notation**, Vol. 1 by Cajori.

b. Squaring a binomial: $(a + b)^2$; $(a + b)(a - b)$; etc.

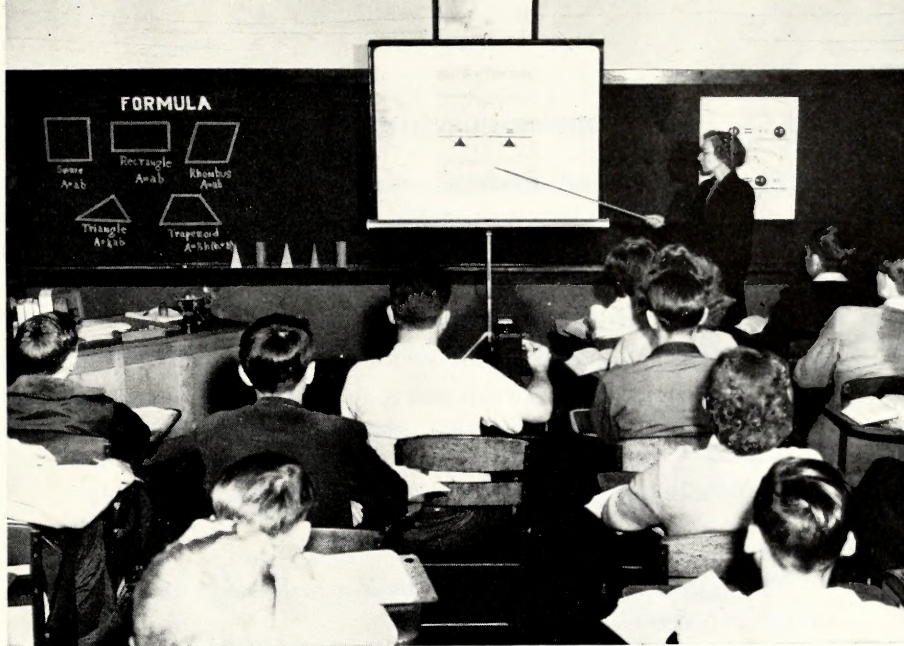
c. Equations with simple algebraic fractions: $\frac{x+2}{3}=7$, etc.

14. Establishment of relationships between table, formula, equation, and graph. See examples in 10 and 11 above.
15. Solution of first degree equations involving one unknown quantity using the following axioms: If equals are divided by equals the quotients are equal; if equals are multiplied by equals the products are equal; if equals are subtracted from equals the differences are equal; if equals are added to equals the sums are equal.
16. Translation of statements into formulas. Example: The arm strength of boys up to age 17 is determined by adding the pull-ups (chins) to the push-ups (as on parallel bars) and multiplying this sum by the result of taking $1/10$ of the boy's weight in pounds plus the height in inches minus 60. In formula this is $A = (P + p)(w/10 + h - 60)$. In physical education this affords a means of classifying boys as to athletic powers.
17. Use of equations in the solution of problems arising in home, business, shop, science, etc.
18. Problems involving equations applied to measurements of perimeters, areas of squares and rectangles, triangles, circles, volumes and surfaces of the rectangular prism, triangular prism, cylinder, etc.
19. Scale drawing. Making maps of school grounds; making maps of the State showing by counties, population, production of agriculture, production in industry, and school population; learning to read blueprints. .
20. Oral and written reports on special topics chosen by pupils because of individual interests.

Course B: First Year Algebra.

This course is designed for those students who should begin the study of algebra in the first year of high school. For those pupils who take Course A in the first year, Course B may be offered as an elective in the second year.

1. Inventory test on skills and principles related to equations and formulas. Review and extension of concepts of coefficient, equation, formula, similar terms.



Using film strip as an aid to teaching

2. Story of the development of algebra.¹
3. Practice in the expression of quantitative statements in algebraic shorthand.
4. Evaluation of formulas and algebraic expressions.
5. Development of concepts and skills in the use of base, exponent, power, and laws of exponents.
6. Interpretation and application of common formulas relating to areas, volumes, etc.
7. Making of formulas based on verbal statements.
8. Making of formulas based on tables.
9. Solution of simple linear equations involving only one unknown, using the following axioms: If equals are divided by equals the quotients are equal; if equals are multiplied by equals the products are equal; if equals are subtracted from equals the differences are equal; if equals are added to equals the sums are equal.
10. Use of equations to solve problems involving percent, simple interest, other business problems of discount, commission, profit and loss, taxation.
11. Review and extension of ability to make and interpret statistical graphs (bar, line, circle, picture, map etc.) ; graphs of formulas.

¹See *The History of Mathematics* by Sanford.

12. Graphical and algebraic solutions of pairs of first degree equations in two unknowns.
13. Problems involving two first degree equations in two unknowns.
14. Special products and factoring.
15. Fractions and fractional equations including decimals.
16. Relationships between table, formula, equation, and graph.
17. Oral and written reports on special topics.

MATHEMATICS FOR YEARS X—XII (Elective)

The courses in mathematics for Years X—XII are designed to meet two specific needs:

1. To equip students who are not going to college to meet practical everyday problems of a quantitative type. For this group two courses are offered, (1) a course in business arithmetic with the emphasis upon arithmetical and graphical study of economic, social, and industrial problems, and (2) a course in basic mathematics covering the practical applications of arithmetic, informal geometry, algebra and trigonometry.
2. To prepare students to take mathematics in college and also for those who desire to continue the study of mathematics in high school, and therefore want the more formal type of courses. For this group any or all of the courses outlined may be offered. Students preparing for college should take at least second year algebra, plane geometry and if possible the course in advanced algebra. Students preparing to enter engineering or technical schools should add to their electives solid geometry and trigonometry.

TENTH YEAR

Second Year Algebra (Elective), 1 year.

1. Inventory test of skills and principles studied in first year algebra. Review of fundamental operations, with algebraic expressions as needed. Emphasis on signed numbers.
2. Review the techniques necessary for the evaluation and solution of formulas and equations of the first degree. Emphasis on checking.

3. Fractions and fractional equations, including decimals. Problems on motion, work, etc.
4. Ratio, proportion, and variation with application to indirect measurement, blue-prints, recipes, enlarging maps, etc.
5. The study of the four-fold relationships in connection with tables, formulas, graphs, and equations. Use squared paper.
6. Fundamental laws of exponents, square root, radicals, fractional exponents, negative exponents, imaginary numbers, equations involving fractional exponents, use of tables of logarithms.
7. Quadratic equations in one unknown. Graph of quadratic function, showing maximum and minimum values, with applications.
8. Simultaneous equations: both of first degree; one first and one second degree. Problems related to motion, money, space, etc.
9. Arithmetic and geometric series with problems.
10. Story of algebra.¹
11. Oral and written reports on topics chosen by pupils because of special interests.

ELEVENTH YEAR

Course A: Plane Geometry (Elective), 1 year.

1. Inventory test for concepts, skills, and clarity of expression, followed by review of construction work using ruler, compasses, and protractor; and re-establishment of concepts and principles relating to basic figures.
2. Establishment of the working principles consisting of definitions, postulates, axioms, and undefined terms.
3. Development of an understanding of what is meant by a proof, different types of proof, both non-mathematical and mathematical.
4. Development of skill in demonstration of theorems.
5. Facts and principles related to such topics as congruence, similarity, concurrence, parallelism, indirect measurement, mensuration, loci, and construction.
6. A wide range of applications for the method of postulational thinking, as in athletic contests, court room, etc.

¹See *History of Mathematics* by Sanford.



Geometry is related to Safety Education

7. Construction and use of models and instruments, such as pantograph, proportional compasses, etc.
8. Applications of geometry to designing, art, architecture, crafts, mechanical drawing, map making, etc.
9. Story of geometry; builders of geometry; discussion of some features of modern geometry, such as Theorem of Menelaus, nine point circle theorem, etc.
10. Oral and written reports on topics chosen by pupils because of special interests.

Course B: Plane and Solid Geometry, Integrated (Elective) 1 year.

1. Basic propositions with lines, planes, and solids.
(Standard lists of propositions as found in reports of National Commission on Mathematical Requirements, College Entrance Board, Regents of the University of the State of New York, etc.)
2. Free hand sketches of two- and three-dimensional figures and constructions using compass, ruler, and protractor.
3. Student-made models and instruments.
4. Correlation of work in geometry through mensuration to arithmetic and algebra.
5. Development of greater skill in using deductive reasoning as related to geometric ideas and as found in life situations.
6. Applications of geometry to designing, art, architecture, crafts, mechanical drawing, map making, etc.

7. Story of geometry; builders of geometry; discussion of some features of modern geometry such as the nine point circle theorem, Theorem of Menelaus, etc.
8. Oral and written reports on topics chosen by pupils because of special interests.

TWELFTH YEAR

Course A: Business Arithmetic (Elective) 1 year.

(This course to be taken in either the Eleventh or Twelfth Year.)

1. Inventory test for concepts, skills and accuracy followed by a restudy of the fundamental processes with common and decimal fractions and per cents. Use problem situations.
2. Short-cut methods of computation.
3. Understanding and use of elementary statistical methods:
 - a. "Gallup Poll," athletic records, reports of insurance companies, etc.
 - b. Some technical skill in making accurate charts and graphs to express statistical data. Examples: Population (census figures), labor, production. State and county maps showing population, production, etc.
 - c. Frequency tables, scatter diagrams, arithmetic average, median.
4. Use of tables: square root, logarithm, interest, investment, insurance.
5. Investments, such as building and loan, stocks, bonds, mortgages, banking procedures, cost of owning a home, annuities, other forms of thrift.
6. Advantages and disadvantages of installment buying.
7. Family budgets.
8. Base of taxation and bond issues. Common forms of taxes.
9. Demonstration of use of computing machines.
10. Construction and use of index numbers (wages, cost of living, etc.)
11. Study of common forms of insurance.
12. National policies relating to price fixing, crop control, tariffs, social security, etc.
13. Measurement of lengths, surfaces, and volumes.

14. Reports on special topics related to the work of the course. (There should be flexibility in this course both as to the content and the method of development of the course. See also Publication No. 267, *Curriculum Guide and Courses of Study in Business Education for North Carolina*, p. 54, "Business Mathematics".)

Course B: Algebra (Elective) One-Half Year.

1. Review and extension of basic concepts, principles and skills.
2. Linear and quadratic functions and equations. Use of graphs.
3. Formulation and solution of problems of commerce, industry, science, home, and community.
4. Exponents, radicals, and radical equations, with problems.
5. Arithmetic, geometric, and binomial series, with problems.
6. Solutions of simultaneous equations of first and second degree.
7. Establishment of the four-fold relationship between formula, graph, equation, and table.
8. Story of the development of algebra,¹ reports on special topics.
9. Reports on symbols, fractions, computation, Vieta, Descartes, Newton, Records, etc.²

Course C: Solid Geometry (Elective) One-Half Year.

1. The basic propositions with the relation of lines and planes in space, and properties and mensuration of solids.
2. Free-hand sketching from student-made models, both solid and wire-outline; tracing of shadows of models.
3. Basic formulas for mensuration: Lengths, areas, and volumes.
4. Correlation of solid geometry through mensuration with arithmetic and algebra.
5. Exhibit simple theorems from Modern Geometry. Examples: Nine point circle theorem, Pascal's theorem on hexagons, etc.³
6. Oral and written reports on topics chosen by pupils because of special interests.

¹See *The History of Mathematics* by Sanford.

²Bell. *Men of Mathematics*; Sanford, *op cit.* Dantzig: *The Language of Science*.

³Sanford. *op cit.*; Cajori. *The History of Mathematics*.

Course D: Trigonometry Elective) One-Half Year.

1. Six trigonometric functions of the general angle.
2. Reduction formulas.
3. Basic identities.
4. Addition formulas for sine, cosine, and tangent.
5. Radian measure.
6. Double-angle and half-angle formulas.
7. Solution of general triangle.
8. Practical problems on heights and distances, and elementary problems in air and sea navigation.
9. Field work using home-made transit and plane table.
10. Make and use slide rule.
11. Story of the development of Trigonometry.

Course E: Basic Mathematics (Elective).

(This course to be taken in either the Eleventh or Twelfth Year.)

This course was devised during the war period to provide a refresher course for students prior to entrance into the Armed Services. It is believed that the course still has validity.

The general purpose of the course is to provide, in the last year or two of secondary school, a study that will have the widest possible usefulness. It should provide a review of knowledges and skills with special emphasis on the practical applications of mathematics. It should include:

ARITHMETIC

A. *Fundamental Operations.*

1. Inventory test to determine which basic skills and techniques need emphasis.
2. Fundamental operations using whole numbers.
3. Reading and writing whole numbers.
4. Denominate numbers.
5. Making and solving problems related to every day affairs.

B. *Common Fractions.*

1. Multiplication and division.
2. Addition and subtraction, mixed numbers.
3. Reduction of fractions, cancellation.
4. Making and solving problems involving fractions.

C. Decimal Fractions.

1. Reading and writing decimals.
2. Adding and subtracting.
3. Multiplying and dividing.
4. Changing common fractions to decimal fractions.
Changing decimal fractions to common fractions.
5. Problems involving decimals, as money matters, measuring.

D. Percentage.

1. Reading and writing per cents.
2. Finding a per cent of a number of quantity.
3. Finding what per cent one number is of another number.
4. Finding a number when a per cent of the number is given
(Concrete illustrations of 2, 3, 4 above).
5. Rounding off numbers, approximations.
6. Problems involving the use of per cent and decimals as in:
discount, commission, interest, other money matters, measuring.

INFORMAL GEOMETRY

A. Direct Measurement.

1. Instruments: ruler, compass, protractor, plane table, steel tape.
2. Measuring lengths, distances and heights.
3. Estimating and checking by measuring, using approximation methods.

B. Scale Drawing.

1. Making and reading maps, enlarging maps.
2. Reading blue prints, using plane table.

C. Circular Measure.

1. Terms related to circle: radius, diameter, chord, circumference, tangent, etc.
2. Angle measures: degree, minute, radian, inscribed and circumscribed polygons, using protractor.
3. Formulas growing out of circular measure.

D. *Triangle.*

1. Terms related to the triangle: base, altitude, median, right angle triangle, acute angle triangle, obtuse angle triangle, circumcenter, in-center.
2. Construction of triangles.
3. Angle sum, area, informal development of formulas.

E. *Area, Volumes, Weights.*

1. Standard units of measure.
2. Relationships between units of measure; examples: 1 cu. ft. of water = 62.5 lbs., 231 cu. in. = 1 gallon.
3. Making estimates and checking by measuring.
4. Developing formulas which grow out of above work.

F. *Symmetry, Congruence, and Similarity.*

1. Drawing figures.
2. Collecting illustrations.

ALGEBRA

A. *Literal Numbers.*

1. Fundamental operations.

A class learns to make and use geometrical figures



2. Evaluation of formulas. Example: $A = \pi r^2$.
Let $r = 3$ inches, then let $r = 6$ inches; compare results.
3. Using monomial and binomial expressions with letters representing numbers.

B. *Graphs.*

1. Read and make line, bar and circle graphs.
2. Using data related to motion, distance, areas, weight, money matters, etc.

C. *Equations.*

1. Concrete illustrations of the equation concept.
2. Solving and evaluating equations.
3. Permissible operations on the equation.
4. Special products, factors.
5. Signed numbers.
6. Short-cut methods.

D. *Exponents and Radicals.*

1. Rules for operation.
2. Square and square root tables.

TRIGONOMETRY

1. Instruments: ruler, compass, protractor.
2. Measuring angles, using protractor, hysometer.
3. Learning and using the trigonometric ratios.
4. Fundamental identities, hypotenuse rule.
5. Computing heights and distances, using tables of logarithms, and natural functions.

In the course outlined above, the approximate percentages of time to be devoted to different phases of mathematics are suggested as follows:

Arithmetic	40%	Algebra	30%
Geometry	20%	Trigonometry	10%

The scope of the foregoing outline of content provides for a two-semester course. Adaptations should be made in the light of individual needs of the pupils enrolled. In the development of this course, it is recommended that major emphasis be placed upon industrial applications. Thoroughness and mastery should be major objectives rather than determination to have each pupil complete a pre-determined schedule of assignments.

Texts for the Course.

There is no one basal book for this course, but there are several which could be used. The following are suggested:

Hart. *Basic Mathematics*. (Brief edition). Heath.

Hart. *Basic Mathematics*. (Complete edition). Heath.

Betz. *Basic Mathematics*. Ginn.

Although these books are not at the present time on the State adopted list, the Division of Textbooks has a number of copies on hand. Some of the schools probably have copies, since this course was offered during the war.

Work in the course should be supplemented by the following books which are on the State supplementary list:

Hausle, Braverman, Eisner, Peters. *Mathematics You Need*. Van Nostrand.

Young. *Rural Arithmetic*. Bruce.

Lasley and Mudd. *The New Applied Mathematics*. Prentice-Hall.

Schorling, Clark and Langford. *Mathematics for the Consumer*. World.

MATHEMATICS AND NATURAL SCIENCE

In the May 1950 issue of School Science and Mathematics there is an article entitled "A Unified and Continuous Program in Mathematics" by Harold P. Fawcett of Ohio State University. In one section of this article he says: "Mathematics is not a set of isolated and unrelated topics. It is a system of ideas unified by a number of fundamental concepts which grow in meaning and significance for the student as his study of mathematics continues. To develop insight and understanding concerning the nature of these concepts is the major responsibility of the mathematics teachers on all levels, while it is also a part of the general responsibility of all teachers regardless of the area with which they may be associated."

One of the fundamental concepts that he discussed was that of measurement. Since it is only by measurement that the forces of nature can be understood and brought under control, then this concept is particularly apropos to the work of science and mathematics teachers. To measure is to know and to have insight, and the story of measurement is the story of man searching for standards that do not vary. In any program of general

$$\text{Formula: } \frac{\text{force on large piston}}{\text{force on small piston}} = \frac{\text{area of large piston}}{\text{area of small piston}}$$

$$\frac{F}{f} = \frac{A}{a}$$

$$f = 10 \text{ pounds}$$

$$A = 50 \text{ sq. in.}$$

$$F = ? \text{ pounds}$$

$$a = 2 \text{ sq. in.}$$

To solve substitute the values in the above formula.

3. A piece of metal weighs 100 grams in air and 88 grams in water. What is its specific gravity?

$$\text{Formula: Specific gravity} = \frac{\text{Weight in air}}{\text{weight in air} - \text{weight in water}}$$

$$\text{weight in air} = 100 \text{ grams}$$

$$\text{weight in water} = 88 \text{ grams}$$

To solve substitute the values in the above formula.

4. Starting from rest, an object falls freely for 3 seconds. How far does it fall?

$$\text{Formula: distance} = \text{one half the acceleration due to gravity times the time squared}$$

$$S = \frac{1}{2}gT^2$$

$$g = 32 \text{ feet per second}$$

$$T = 3 \text{ seconds}$$

To solve substitute the values in the above formula.

5. An automobile weighing 2500 pounds is traveling at the rate of 30 miles per hour (44 feet per second). What is its kinetic energy?

$$\text{Kinetic energy} = \frac{\text{weight}}{\text{two}} \times \frac{\text{velocity squared}}{\text{acceleration due to gravity}}$$

$$\text{K.E.} = \frac{wv^2}{2g}$$

$$w = 2500 \text{ pounds}$$

$$v = 44 \text{ feet per second}$$

$$g = 32 \text{ feet per second per second}$$

To solve substitute the values in the above formula.

6. A certain weight of gas has a volume of 1200 cubic centimeters at a pressure of 500 grams per square centimeter and a temperature of 21 degrees centigrade. What is its volume at a pressure of 1500 grams per square centimeter and a temperature of 315 degrees centigrade?

$$\text{Formula} = \frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

$$P_1 = 500 \text{ grams per square centimeter}$$

$$P_2 = 1500 \text{ grams per square centimeter}$$

$$T_1 = 21^\circ\text{C or } 294^\circ\text{A}$$

$$T_2 = 315^\circ\text{C or } 588^\circ\text{A}$$

$$V_1 = 1200 \text{ cubic centimeters}$$

To solve substitute the values in the above formula.

7. What is the combined resistance of a 9-ohm coil and an 18-ohm coil connected in parallel?

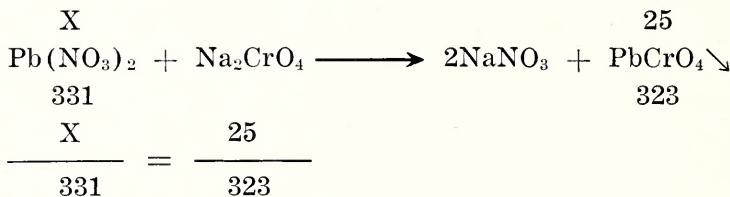
$$\text{Formula: } 1/R = 1/R_1 + 1/R_2$$

$$R_1 = 9 \text{ ohms}$$

$$R_2 = 18 \text{ ohms}$$

To solve substitute the values in the above formula.

8. A paint manufacturer wishes to know how many pounds of lead nitrate, $\text{Pb}(\text{NO}_3)_2$, he will have to add in a water solution of sodium chromate, Na_2CrO_4 , to make 25 pounds of the insoluble chrome-yellow pigment, lead chromate, PbCrO_4 .



A solution of the above equation will give the answer.

10. How many millimeters of 0.4 normal potassium hydroxide solution, KOH, will neutralize 100 millimeters of 0.05 normal phosphoric acid, H_3PO_4 ?

$$\begin{array}{rcl} \text{Formula:} & \frac{\text{millimeters of acid}}{\text{millimeters of base}} & = \frac{\text{normality of base}}{\text{normality of acid}} \\ & \frac{100}{X} & = \frac{0.4}{0.05} \end{array}$$

A solution of the above equation will give the answer.

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